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TFP in Germany and the USA (2000–2017): Classical and Bayesian Approaches to Trend Analysis

Abstract

The study aims to analyse the trends in total factor productivity (TFP) in Germany and the USA between 2000 and 2017, assessing whether productivity growth differs significantly between the two economies. TFP was calculated using the Cobb–Douglas production function and the Solow residual. Trends were estimated through linear and interaction models, while robustness was assessed with nonparametric permutation tests. In addition, a Bayesian panel model was employed to incorporate prior knowledge about the production function and to obtain more stable estimates. Posterior tests were applied to evaluate model fit and the reliability of results. Parametric estimates indicated statistically significantly higher TFP growth in the USA compared to Germany. However, nonparametric permutation tests did not confirm these differences, underlining the limitations of short time series. The Bayesian panel approach provided consistent, robust results, supporting the validity of combining classical and Bayesian techniques.

Despite the short time span and limited sample, the study demonstrates a novel methodological framework for comparative productivity analysis. By combining frequentist and Bayesian approaches, it highlights the potential for more robust inference. It provides a model that can be applied to analyses of other countries or extended time horizons.

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1. Introduction

The analysis of total factor productivity (TFP) is a central point of departure in economic research on long-term economic growth, as it allows separating the effects of capital and labour from those stemming from technological progress, organisational change, and institutional efficiency. Differences in TFP across countries have been the subject of numerous studies in recent decades, with the United States often portrayed as an economy with markedly higher productivity compared to European countries, including Germany (van Ark, Inklaar & McGuckin, 2003; O'Mahony & Timmer, 2009; Gordon, 2016). The reasons for these differences are linked to greater capital accumulation intensity, more flexible markets, and faster innovation diffusion in the U.S. (OECD, 2015; OECD, 2020).

This paper focuses on the period 2000–2017, which is specific to both economies. It marks the time after the technological boom of the 1990s, characterised by the growth of digitalisation and the service sector in the U.S., and the period in which Germany underwent labour market reforms and gradually strengthened the role of its export-oriented industry. A particular challenge in this period is the 2008 global financial crisis, which affected both countries but may have had different impacts on their long-term productivity trends.

The methodological approach is based on a combination of classical and modern tools, applied sequentially and complementarily. First, parametric regression models (OLS) are used to test the relationships between capital, labour, and value added, and to calculate the Solow residual as a measure of TFP. This establishes the standard Cobb–Douglas specification, which is a common reference point in the literature. In the second step, TFP trends are examined separately for each country and via interaction models, testing the hypothesis of slope equality—that is, whether TFP in both countries is increasing at the same rate.

Since the sample size is small ($n = 18$ years), classical parametric tests may not be reliable. We therefore also include a nonparametric permutation test, which is robust to distributional assumptions, though less powerful in small samples. This combination of methods allows us to check, even in the initial phase of analysis, whether the observed differences between Germany and the U.S. are methodologically stable.

However, both the OLS and nonparametric approaches have important limitations. The former relies on asymptotic properties, which do not hold in small samples, while the latter struggles to distinguish signal from noise. This makes the Bayesian approach a natural next step, as it enables the inclusion of prior knowledge about typical shares of capital and labour ($\alpha \approx 0.3$; $\beta \approx 0.65$), while providing a more comprehensive treatment of uncertainty through posterior distributions. Using a Bayesian panel model with random effects, we analyse not only the average trend but also the initial differences between the two countries and the robustness of the results with respect to crisis episodes, such as 2008.

On this basis, the central research question is whether the productivity trends of Germany and the United States during 2000–2017 were the same. The null hypothesis states that the TFP trends are equal, while the alternative hypothesis posits that the U.S. trend is steeper, reflecting faster productivity growth than Germany's.

Nevertheless, the analysis has an important limitation: the available time series covers only 2000–2017, which means the sample is short and sensitive to individual external shocks such as the global financial crisis. This limits the power of nonparametric tests and the statistical confirmation of differences between the two countries. However, the study does not aim solely at definitive empirical conclusions but, above all, at presenting a methodological framework that combines classical (OLS, AR, interaction tests) and modern Bayesian approaches, thereby enabling a more comprehensive analysis of productivity trends.

2. Literature Review

Total factor productivity (TFP) is a central indicator in the empirical literature on long-term differences in countries' economic performance. Since it captures the part of economic growth that cannot be explained by capital and labour inputs, TFP has become a key instrument for understanding structural differences between economies and their convergence. Numerous comparative studies confirm that TFP in the United

States has grown faster than in most European countries over the past few decades, creating a persistent productivity gap.

Empirical evidence consistently shows the U.S. advantage over European economies. Inklaar, Timmer, and van Ark (2008) find that most of the productivity difference originated in the service sector, where the U.S. more rapidly exploited the effects of information and communication technology. Similarly, Calcagnini, Giombini, and Travaglini (2021) demonstrate that long-term TFP trends in the U.S. are more pronounced than in European countries and Japan, with the gaps even widening after 2000.

At the same time, the literature highlights a general slowdown in productivity growth across advanced economies. Byrne, Fernald, and Reinsdorf (2016) discuss whether this represents a real trend or a measurement problem related to digitalisation and the insufficient capture of service-sector innovations. Studies by Fernald (2014) and Cette, Fernald, and Mojon (2016) further confirm that the slowdown began even before the global financial crisis, suggesting structural constraints rather than merely the consequences of crisis shocks. The most recent approaches to TFP measurement include adjustments for capital and labour utilisation rates. Huo, Levchenko, and Pandalai-Nayar (2023) show that differences in factor utilisation can significantly affect international productivity comparisons.

Of particular interest is the comparative analysis by Bengoa, Pérez, and Fernández (2015), which combined generalised least squares methods with the Kalman filter to examine the period 1950–2001. The results of the first model suggested that U.S. researchers contributed more to pushing out the technological frontier, i.e., to TFP growth, than their German counterparts. The alternative specification using the Kalman filter, however, indicated that the productivity gap between the U.S. and Germany was narrowing in the long run, with Germany even overtaking the U.S. These contrasting results underscore that the assessment of TFP differences is highly dependent on the methodological framework and the definition of the technological frontier.

Recent research has made significant contributions to the understanding of total factor productivity (TFP) dynamics at both the aggregate and sectoral levels. Comin, Gonzalez, Schmitz, and Trigari (2020) developed a new methodology for estimating TFP growth that accounts for non-zero profits and incorporates survey-based proxies for unobserved changes in factor utilisation. Applying their method to European data, including Germany, they found that U.S. TFP growth series are substantially less volatile and often less cyclical compared to those obtained with standard methods. Based on this approach, the authors produced the first quarterly TFP growth series, adjusted for utilisation, for European countries, thereby filling an important data gap. For the United States, its method shows not only lower cyclicality but also a significantly higher average TFP growth rate over the period 1988–2020.

Crafts and Mills (2017) analysed TFP growth in the U.S. business sector over the past five decades using an unobserved components model, where the trend growth is represented as a stochastic process and deviations as autoregressive noise. Their results indicate a gradual decline in trend TFP growth, from around 1.5% per year in the 1960s to approximately 1% per year by 2016. Moreover, the authors examined different approaches to forecasting long-term TFP trends, showing that forecasts based on the most recent 20–25 years of data suggest even lower growth rates (0.5–0.7%). They further emphasised that TFP growth over 10-year periods is highly unstable, with episodes of strong growth alternating with periods of stagnation. Thus, the current weak TFP performance does not necessarily imply permanently low future growth.

At the sectoral level, Germany has experienced a marked deterioration in allocative efficiency. Surray (2020), in her master's thesis, finds that structural inefficiencies in resource allocation have intensified over the past decade, with the COVID-19 pandemic accelerating pre-existing weaknesses. Using harmonised firm-level microdata and three analytical approaches—the Hsieh and Klenow misallocation measure, the Olley–Pakes decomposition, and Raval's surplus estimates—she demonstrates that frictions in capital allocation have increased and the ability to reallocate resources has diminished in German industry. In particular, the decline of the covariance term in the Olley–Pakes decomposition indicates a weakening of reallocation, as less productive firms continue to maintain market shares. At the same time, new entrants face barriers to growth. The rising dispersion of revenue-based TFP (TFPR) and increased distortions in the marginal revenue

product of capital (MRPC) further suggest that Germany's main challenge lies not in a lack of technological potential, but in the inefficient use of existing resources.
Methodologically, the Bayesian framework has proven particularly useful for analysing growth and productivity factors. Moral-Benito (2012) shows that Bayesian panel models provide more stable estimates than classical approaches, especially in cases with a limited number of observations, because they allow the incorporation of prior information about parameter structure. A similar approach is relevant for analysing TFP differences between Germany and the U.S., where the available time horizon is relatively short.
Taken together, the literature confirms two key facts: (i) TFP is the central factor explaining long-term differences in economic growth, and (ii) U.S. TFP trends are more pronounced than European ones, with Germany being a characteristic example of slower dynamics. However, open questions remain: whether this reflects structural constraints in European economies or methodological challenges in productivity measurement. The Bayesian framework, which integrates economic theory and empirical data, represents an appropriate methodological advancement for addressing these questions.

3. Data and Methodology

The analysis of productivity is based on the Cobb–Douglas production function, which is frequently used in the economic literature as a starting point for studying the contribution of capital and labour to economic growth. Its advantage lies in the straightforward interpretation of parameters, as these directly represent the elasticities of output with respect to inputs. Within this framework, the central object of analysis is total factor productivity (TFP), interpreted as the residual—the part of economic growth that cannot be explained by capital accumulation or increases in employment. TFP is closely linked to technological progress, organisational change, and other efficiency-related factors; therefore, comparing its trends across countries provides insight into differences in development. The research question focuses on Germany and the United States in the period 2000–2017, with the aim of examining whether TFP trends in the two countries diverge.
The first step is the estimation of the Cobb–Douglas production function using ordinary least squares (OLS). This step is necessary because it provides a baseline estimate of the elasticities of capital and labour and allows us to verify whether the data are consistent with the assumed functional form. The log-linear regression model for country *i* at time *t* can be written as:

$$lnY_{it} = \beta_0^i + \alpha^i lnK_{it} + \beta^i lnL_{it} + \varepsilon_{it} \quad (3.1.)$$

where lnY_{it} is the logarithm of GDP, lnK_{it} capital, lnL_{it} labour, β_0^i constant, α^i β^i the output elasticities and ε_{it} the error term. This provides the parameters that serve as the basis for calculating the TFP residuals and, at the same time, as a reference for selecting prior distributions in the Bayesian analysis. The OLS results thus represent the first insight into the hypothesis concerning differences between the two countries.
Since OLS does not provide insight into the dynamic structure of productivity, in the next step, we estimated first-order autoregressive models (AR(1)):

$$TFP_{i,t} = \gamma_0^i + \gamma_1^i TFP_{i,t-1} + u_{i,t} \qquad (3.2.)$$

Where $TFP_{i,t}$ denotes the total factor productivity of country i at time t, γ_0^i and γ_1^i are parameters capturing the dynamics of the process, and $u_{i,t}$is the random disturbance.
These models allow us to test whether TFP in a given country exhibits persistence, i.e., whether it follows a stationary process. This provides insight into the dynamics of TFP and a better understanding of whether the observed movements are merely short-term fluctuations or systematic trends. Moreover, the AR(1) results complement the OLS findings and provide a basis for formulating the hypothesis about differences in trends between the two countries.
To test this hypothesis directly, we constructed trend models with an interaction term:

$$TFP_{it} = \beta_0 + \beta_1 \cdot t + \beta_2 \cdot USA_i + \beta_3 \cdot (t \cdot USA_i) + \varepsilon_{it} \quad (3.3.)$$

Where *t* is the time variable, USA_i takes the value 1 for the United States and 0 for Germany, and ε_{it} is the error term. The coefficient β_1 captures the long-term trend for Germany, while the coefficient β_3 reflects the difference in trend between Germany and the United States. Testing the hypothesis H_0: $\beta_3 = 0$ directly addresses the research question of whether systematic differences in TFP trends exist. The interaction models were tested using both classical parametric tests and a nonparametric permutation test, the latter being less sensitive to assumptions about error distributions. The results showed that parametric methods consistently detected a difference, while the nonparametric approach highlighted the limitations of the small sample size.

In this context, the shift to the Bayesian approach is particularly well justified. The sample consists of only 36 observations, limiting the power of classical tests and increasing uncertainty. The Bayesian panel model makes it possible to incorporate prior knowledge about the parameters of the Cobb–Douglas function, improving the stability of estimates, while at the same time providing complete posterior distributions that capture uncertainty comprehensively. The panel specification with random effects by country can be written as:

$$lnY_{it} = \alpha_i + \beta_k lnK_{it} + \beta_L lnL_{it} + \gamma \cdot t + \delta \cdot (t \cdot USA_i) + u_{it} \quad (3.4.)$$

Where α_i captures the country-specific random effects, γ represents the common time trend, and δ is the interaction term measuring the difference in the trend for the United States. The prior distributions were chosen in line with theory and empirical research: a normal prior for capital $N(0.3, 0.05^2)$, a normal prior for labour $N(0.65, 0.05^2)$, weaker normal priors for the time trend and interaction $N(0,0.01^2)$, and an exponential prior for the error variance.

The model was estimated using the Hamiltonian Monte Carlo (HMC) method, which enables efficient sampling even in small samples. To assess model adequacy, we employed diagnostic indicators (Rhat, effective sample sizes) and a posterior predictive check; in addition, for model specification comparison we used the Leave-One-Out (LOO) criterion. Particular attention was also devoted to residuals and robustness: the distributions of residuals were examined, and the sensitivity of results to the exclusion of the crisis year 2008 was tested, ensuring additional reliability in the interpretation of results.

For the empirical analysis of total factor productivity (TFP) in the period 2000–2017, we use data for Germany and the United States, fully drawn from the Penn World Table, version 10.01 (Groningen Growth and Development Centre – GGDC). Real gross domestic product (Y) is expressed in international dollars at purchasing power parity (PPP) and constant 2017 prices, which allows for temporal and cross-country comparability. The capital stock (K) is measured in international dollars at current PPP and estimated using the perpetual inventory method, providing a consistent measure of capital accumulation. Labour (L) is captured as the average number of hours worked, which better reflects labour input intensity than the mere number of employees. PWT 10.01 provides harmonised time series on income, production, inputs, and productivity for 183 countries over the period 1950–2019, enabling reliable comparison between Germany and the United States (Feenstra, Inklaar & Timmer, 2015).

The analysis is based on data from the Penn World Table, version 10.01, covering the period 2000–2017. The choice of the period is not arbitrary but reflects the availability of data at the time of preparing the bachelor's thesis, which serves as the basis for this article. Accordingly, the final year of the sample corresponds to the latest year available in the database at the time of data collection.

3. Results

Descriptive Statistics

A basic descriptive overview of the data, transformed into logarithms and summarised in Table 1, confirms the differences in scale between the German and U.S. economies during 2000–2017. The level of log-transformed value added in Germany is lower, reflecting its smaller absolute GDP, while the values for the U.S. are significantly higher, consistent with the expected size of its economy. Capital in both countries grew steadily throughout the period, but the levels were considerably higher in the U.S., indicating more intensive capital accumulation and greater potential for productivity effects. In Germany, investment dynamics were more moderate, reflected in a lower, narrower range of capital.

Labour data in both countries are relatively stable, with only minor annual fluctuations. This implies that value growth added during the analysed period did not stem from changes in the size of the labour force, but primarily from capital accumulation and changes in the efficiency of production factors. In Germany, labour force dynamics were somewhat more even, while in the U.S., despite higher absolute levels, the variation was similarly narrow. Such a structure suggests that the Solow residual, i.e., total factor productivity (TFP), is the key variable for explaining the long-term differences in economic trends between the two countries.

Table 1: Descriptive statistics of log-transformed series (2000–2017) (values shown as minimum, quartiles, median, mean, maximum)

Variable	Min	1st quartile	Median	Average	3rd quartile	Max
lnGDPg	14.92	14.95	15.06	15.06	15.13	15.26
lnKg	16.19	16.24	16.49	16.47	16.67	16.77
lnLg	3.553	3.562	3.574	3.585	3.607	3.636
lnGDPusa	16.46	16.56	16.62	16.62	16.68	16.80
lnKusa	17.71	17.82	17.91	17.89	17.96	18.02
lnLusa	3.539	3.554	3.563	3.566	3.574	3.604

Since TFP is defined as a residual that captures the difference between actual growth and the contributions of capital and labour, it is important to examine the properties of the series from which it will be calculated. The results of the unit root test, presented in Table 2, confirm that all the underlying series in levels are non-stationary. The augmented Dickey–Fuller test does not reject the null hypothesis of a unit root for most variables, indicating that GDP, capital, and labour in both countries follow trend processes. The KPSS test complements this picture, as its values reject the hypothesis of stationarity around a constant. The DF–GLS test, as a stronger alternative, also fails to confirm stationarity, since the obtained statistics remain above the critical thresholds.

Table 2: Results of stationarity tests

Series	ADF statistics (p-value)	KPSS statistics	DF-GLS statistics	Conclusion
lnGDPg	–2.79 (0.27)	0.699	0.753	Non-stationary
lnKg	–1.20 (0.88)	–	–	Non-stationary
lnLg	–0.93 (0.93)	–	–	Non-stationary
lnGDPusa	–2.70 (0.31)	–	–	Non-stationary
lnKusa	–3.33 (0.09)	–	–	Non-stationary
lnLusa	–0.88 (0.94)	–	–	Non-stationary
tfp_g	–	0.699	0.753	Non-stationary
tfp_usa	–	0.679	0.734	Non-stationary

Note: KPSS and DF–GLS are reported for the TFP series for which the tests were performed. Critical values: ADF (–2.71 at 1%, –1.96 at 5%), KPSS (0.463 at 5%), DF–GLS (–1.96 at 5%).

OLS Estimates and Diagnostic Tests

The regression results, summarised in Table 3, confirm the existence of a stable long-term relationship between GDP, capital, and labour, consistent with the assumptions of the Cobb–Douglas production function. For Germany, capital is found to have a statistically significant and positive effect on GDP, while the contribution of labour is not statistically significant. The estimated coefficient on capital is 0.51, meaning that a one-percent change in the capital stock increases GDP by about half a percent. The coefficient on labour (0.22) is not statistically significant, suggesting that during the period 2000–2017 the size of the labour force in Germany was not a primary driver of aggregate value-added growth.

For the United States, the picture is quite different. The estimated coefficients for capital (1.37) and labour (2.36) are very high and statistically significant. This confirms a strong relationship between capital accumulation, labour force growth, and economic growth in the U.S. economy, which can also be interpreted as the result of greater technological absorption and organizational change. However, the estimated values are considerably higher than the standard parameters of the Cobb–Douglas function ($\alpha \approx 0.3$, $1-\alpha \approx 0.7$), indicating that this represents a cointegration relationship reflecting the long-term association between the variables rather than a direct production function in the narrower sense.

Table 3: OLS estimates of the Cobb–Douglas function (2000–2017)

Country	Intercept	lnK (coeff.)	lnL (coeff.)	R^2	Adj. R^2	Shapiro-Wilk (p)	JB test (p)	BP test (p)
Germany	5.90 (0.10)	0.51*** (0.00)	0.22 (0.73)	0.949	0.943	0.724	0.757	0.154
USA	–16.38*** (0.00)	1.37*** (0.00)	2.36*** (0.00)	0.993	0.992	0.031	0.279	0.110

*Note: p-values are reported in parentheses. Significance codes: *** $p < 0.01$.*

Diagnostic tests support the adequacy of the estimates. For Germany, the Shapiro–Wilk and Jarque–Bera tests confirm the normality of residuals, and no heteroskedasticity was detected. For the United States, the Shapiro–Wilk test indicates a deviation ($p = 0.031$), but the Jarque–Bera test does not confirm it, suggesting that the deviation is minor and does not significantly affect the validity of the model. The Breusch–Pagan test in both regressions does not reject the hypothesis of homoskedasticity, which increases confidence in the results.

The high R^2 values and the stability of the estimates confirm the long-term interdependence between GDP, capital, and labour. This is consistent with the results of the stationarity tests in Table 2 and indicates that the series are cointegrated. Consequently, the use of the Solow residual as a measure of total factor productivity is justified, as it captures the portion of economic growth that cannot be explained by capital and labour.

To further illustrate the estimated relationships between GDP, capital, and labour, we combined the four scatter plots into a single multi-panel figure with regression lines (figure 1). This joint representation highlights several important patterns. For Germany, the positive association between capital and GDP is clear and consistent with the regression results, while labour exhibits a negative slope, which mirrors the non-significant labour coefficient in the OLS model. For the United States, capital and GDP are almost perfectly collinear, while the labour–GDP relationship is unexpectedly negative in the bivariate setting, despite being positive and significant in the multivariate regression.

These inconsistencies point to two technical issues. First, the very strong correlation between capital and labour implies multicollinearity, which distorts the bivariate slopes and inflates the variance of OLS estimates. Second, the small sample size (18 annual observations per country) reduces the power of classical inference and makes parameter estimates highly sensitive to single observations. In such circumstances, scatter plots with regression lines may exaggerate or mask the true structural relationships.

The Bayesian panel framework directly addresses these limitations. By pooling information across countries, it increases statistical efficiency. By incorporating prior distributions on the capital and labour shares, it regularises the estimates and mitigates the effect of multicollinearity. Finally, Bayesian inference provides full posterior distributions rather than point estimates, which means that uncertainty around the productivity trends can be assessed more comprehensively. For these reasons, the transition to the Bayesian model is not only natural but also technically necessary to obtain stable and economically meaningful estimates of TFP dynamics.

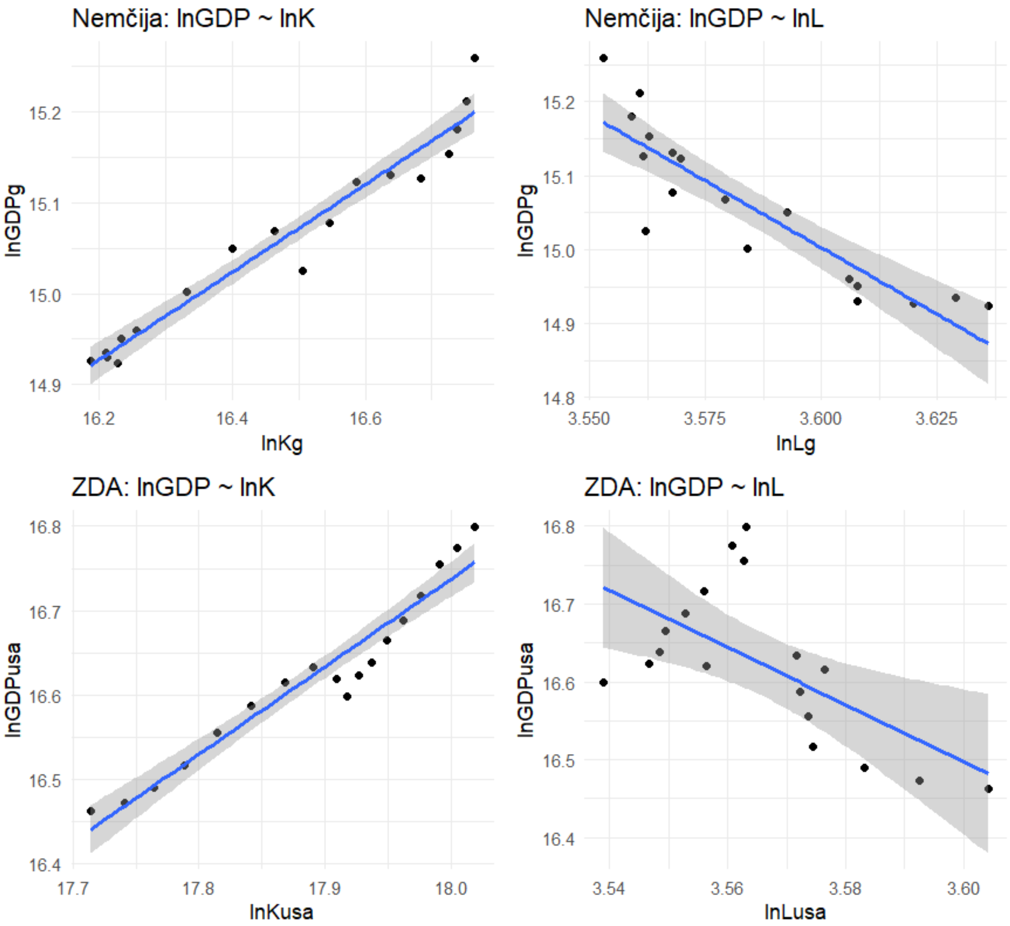

Figure 1: Scatterplots of GDP against capital and labour for Germany and the USA

Analysis of TFP Trends – Parametric and Nonparametric Estimates

We now move from the estimates of the Cobb–Douglas function and the construction of the Solow residual to the central question: do the long-term trends of total factor productivity (TFP) differ between Germany and the United States? The analysis is conducted in two steps. First, we apply parametric methods based on regression trend models and interaction terms, where we test the null hypothesis that the slopes of the trends are equal. We then verify the results with a nonparametric permutation test, which relies on less strict assumptions but has lower power with small samples. In this way, we combine the advantages of both approaches and test the robustness of the conclusions.

We begin by estimating linear models separately for each country, using different values of the capital share α. The results, shown in Table 8, indicate that both economies exhibit a positive and statistically significant TFP growth trend, but the slope is consistently higher in the U.S. (≈0.0137–0.0147) than in Germany (≈0.0088–0.0110). The explained variance is very high in both countries, but greater in the U.S., confirming that American TFP grew faster and more steadily.

In the next step, we formally test the differences with an interaction model that includes the product of the time trend and the U.S. indicator. The results in Table 9 show that the interaction coefficient (Years × USA) is positive and statistically significant in all cases ($p < 0.01$). This means that the TFP growth trend in the U.S. is steeper than in Germany. F-tests of the linear hypothesis consistently reject the null of equal slopes, with the difference between the two countries increasing with higher α: from 0.0037 per year ($\alpha = 0.30$) to

0.0049 per year (α = 0.35). Parametric methods therefore clearly confirm the existence of differences in productivity trends.
In the third step, we verify the results with a nonparametric permutation test (Table 10). Although the observed interaction coefficients are the same as in the regression analysis, the permutation p-values are high (≈0.93–0.95), and the confidence intervals are wide and include zero. This means that the permutation test does not detect the differences between countries as statistically significant. The main reason for this lies in the limited number of observations (n = 18 years), which reduces the power of the permutation approach, whereas the parametric models exploit the structure of the data and thus detect a difference.

Table 4: Results of parametric and nonparametric tests of equality of TFP trends between Germany and the United States (2000–2017)

α	Germany: slope (Years)	USA: slope (Years)	Interaction (USA × Years)	F-test (p)	Perm. coeff.	Perm. p-value
0.30	0.0110***	0.0147***	0.0037**	8.45 (0.0066)	0.0037	0.9452
0.33	0.0097***	0.0141***	0.0044**	11.63 (0.0018)	0.0044	0.9318
0.35	0.0088***	0.0137***	0.0049***	13.87 (0.0008)	0.0049	0.9320

*Note: *** $p < 0.01$; ** $p < 0.05$.*

The table clearly presents a consistent picture: parametric methods (trend and interaction tests) confirm the higher TFP slope in the U.S., while the permutation test does not confirm the differences as statistically significant.
The estimates obtained so far, based on OLS and AR models, have shown that the Cobb–Douglas specification adequately captured the relationship between capital, labour, and economic growth, as the results indicated a high explanatory power of the model and residual stability. The AR(1) approach further revealed a high degree of autoregressive persistence in TFP, confirming that productivity is largely reproduced from its own past values. Trend estimates clearly showed positive dynamics in both countries, with growth rates higher in the U.S. than in Germany, which was also confirmed by the interaction regression. Although the permutation tests highlighted methodological limitations and the small number of observations, the results of classical methods consistently point to a difference in favour of the U.S.
It is precisely at this point that the transition to Bayesian analysis becomes meaningful. The Bayesian approach makes it possible to incorporate prior knowledge about the parameters of the production function, which is crucial in the case of a limited sample, and provides a more comprehensive treatment of uncertainty through posterior distributions. In this way, we obtain not only point estimates but also probability distributions for the parameters and their interactions. Based on the OLS and AR results, we can expect that the Bayesian analysis will confirm positive TFP trends in both countries while at the same time offering a more robust picture of the differences in productivity growth between Germany and the U.S.

Bayesian Panel Model with Interaction

The estimates obtained so far from OLS and AR models have shown that the Cobb–Douglas specification adequately captures the relationship between capital, labour, and economic growth. The results confirmed the high explanatory power of the model and the stability of residuals, while the AR(1) approach revealed strong persistence in TFP, meaning that productivity is largely reproduced from its own past values. Trend estimates indicated that TFP growth was positive in both countries but stronger in the U.S., which was also confirmed by the interaction tests. Although the permutation tests highlighted the limitations of the small sample, the methods used thus far consistently suggest a difference in favour of the U.S.

At this stage, the transition to Bayesian analysis becomes meaningful. With a small sample such as that for 2000–2017, incorporating prior knowledge about the parameters of the production function is essential for more stable estimates. The Bayesian framework makes this possible through prior distributions, while also providing a more comprehensive treatment of uncertainty via the posterior distributions of the parameters. The results in this section are based on the Bayesian panel model presented in the methodological chapter (see equation 3.4). The model is grounded in the Cobb–Douglas specification, extended with an interaction term that allows for the direct testing of differences in TFP trends between Germany and the U.S. A central role is played by the parameter δ, which measures the difference in slopes between the two countries, with a positive value indicating faster TFP growth in the U.S.

The choice of priors was aligned with economic theory and prior empirical studies. For capital, a normal prior with mean 0.3 and standard deviation 0.05 was used; for labour, a normal prior with mean 0.65 and the same standard deviation. This reflects the standard distribution of value added in the Cobb–Douglas function while allowing flexibility in the posterior estimates. For the time trend and its interaction with the U.S., weaker normal priors with mean zero and standard deviation 0.01 were applied, ensuring the model remains sensitive to small but economically meaningful trend differences. For the error variance, an exponential prior was chosen to ensure positivity and stable estimation.

Estimation was conducted within the Bayesian framework using the Hamiltonian Monte Carlo (HMC) algorithm, implemented via the *brms* package through *cmdstanr*. HMC enables efficient sampling from posterior distributions even in multidimensional models and provides accurate estimation of parameter uncertainty. The results are posterior distributions of the coefficients, which provide not only point estimates but also confidence intervals and distributional shapes, offering a richer interpretation of TFP differences between Germany and the U.S.

The results, summarized in Table 4, are consistent with economic theory. The posterior estimate of the capital coefficient is 0.30 and of labour 0.66, almost perfectly matching the prior values and confirming the appropriateness of the methodological design. The time trend is positive (0.01), indicating long-term growth on average. The interaction between year and the U.S. is estimated close to zero, with a confidence interval between 0.00 and 0.01, meaning that in the period 2000–2017 there was no statistically significant difference in trends, although the posterior distribution allows for the possibility of a slightly faster U.S. trend. The residual variance is low ($\sigma = 0.02$), confirming the good fit of the model.

Overall, the Bayesian panel model confirms that productivity trend differences between Germany and the U.S. remained small during the analyzed period. This is consistent with findings in the literature pointing to a general slowdown in productivity growth in advanced economies after 2000, with the U.S. advantage persisting but not substantially increasing.

Table 4: Posterior estimates of the Bayesian panel model with interaction

Parameter	Estimation	Std. Error	95% CI Lower	95% CI Upper
Intercept	–13.99	4.65	–22.23	–5.29
lnK	0.30	0.04	0.21	0.38
lnL	0.66	0.05	0.57	0.76
Years	0.01	0.00	0.01	0.01
countryUSA	–6.67	6.64	–18.09	4.19

Parameter	Estimation	Std. Error	95% CI Lower	95% CI Upper
Years × countryUSA	0.00	0.00	0.00	0.01
σ (residual SD)	0.02	0.00	0.02	0.03

Although the basic results are consistent with economic expectations, it is necessary to further test convergence of chains, stability of posterior distributions, and predictive power of the model in order to assess their reliability. The following section is therefore devoted to validation and robustness checks of the Bayesian panel model.

Validation and Goodness of Fit

To assess the reliability of the Bayesian panel model estimates, we employed a combination of quantitative indicators and graphical validations. The purpose of validation was to determine whether the model adequately captures the structure of the data, whether the results are stable, and whether the posterior distributions are consistent.

The results of the LOO calculation, shown in Table 5, confirm that the model fits the data well. The estimated expected log predictive density (elpd_loo = 88.6) with a low standard error (3.8) indicates high predictive ability. The complexity index (p_loo = 4.3) is moderate and corresponds to the number of estimated parameters, meaning that the model is not overfitted. Pareto k values (< 0.7) are within acceptable limits, confirming that individual observations are not excessively influential. The LOOIC (–177.2) is a low value, further confirming good predictive adequacy.

The Bayesian R^2, estimated at 0.999 with a narrow confidence interval, confirms the model's excellent fit. Importantly, even after including the time trend and the interaction term, the model retains a stable structure, meaning that the priors did not artificially inflate the fit. The random effects by country confirm that the initial differences between Germany and the U.S. were not statistically significant, consistent with the results for the interaction coefficient in Table 4.

Table 5: Fit and validation indicators of the Bayesian panel model

Indicator	Estimation	Std. Error	95 % Confidence Interval
elpd_loo	88.6	3.8	–
p_loo	4.3	1.3	–
looic	–177.2	7.7	–
Bayesian R^2	0.999	0.00005	0.9993 – 0.9995
Random effect (DE)	–0.23	3.23	–6.19 – 5.06
Random effect (USA)	0.16	3.10	–5.26 – 5.83

Note: elpd_loo (expected log predictive density) measures the predictive adequacy of the model – higher values indicate better fit. p_loo represents the effective number of parameters, i.e., model complexity. looic (leave-one-out information criterion) is a measure of fit quality, where lower values indicate a better model. The Bayesian R^2 is the analogue of the classical R^2 and shows the proportion of explained variability, with values close to 1 indicating excellent fit. Random effects capture differences in the initial intercepts between countries.

The first step in validating the Bayesian model is to check chain convergence and the shape of posterior distributions. Figure 2 shows the trace plots for all key model parameters. We can see that all chains mix well, without visible trends or long-lasting deviations, confirming stable convergence. Red divergence markers are rare and localized, and therefore do not affect the quality of sampling.

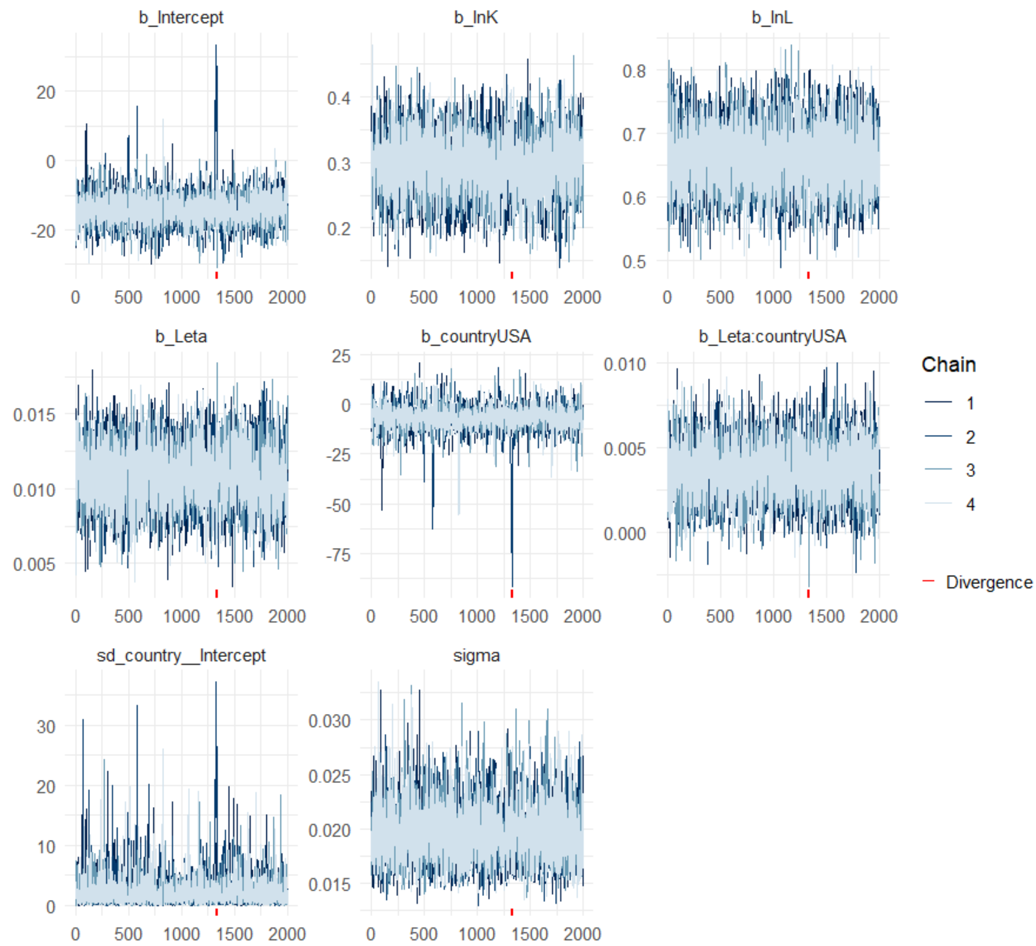

Figure 2: Trace plots of the parameters of the Bayesian model

Figure 3 shows the posterior densities of the same parameters. The distributions are smooth, unimodal, and virtually identical across chains, confirming well-defined posterior distributions. The capital coefficient (lnK) is centred around 0.3, the labour coefficient (lnL) around 0.65, which is consistent with the theoretical expectations of the Cobb–Douglas function. The time trend (Years) has a narrow positive distribution, while the interaction parameter (Years × USA) indicates a small but positive shift in the trend in favor of the U.S.

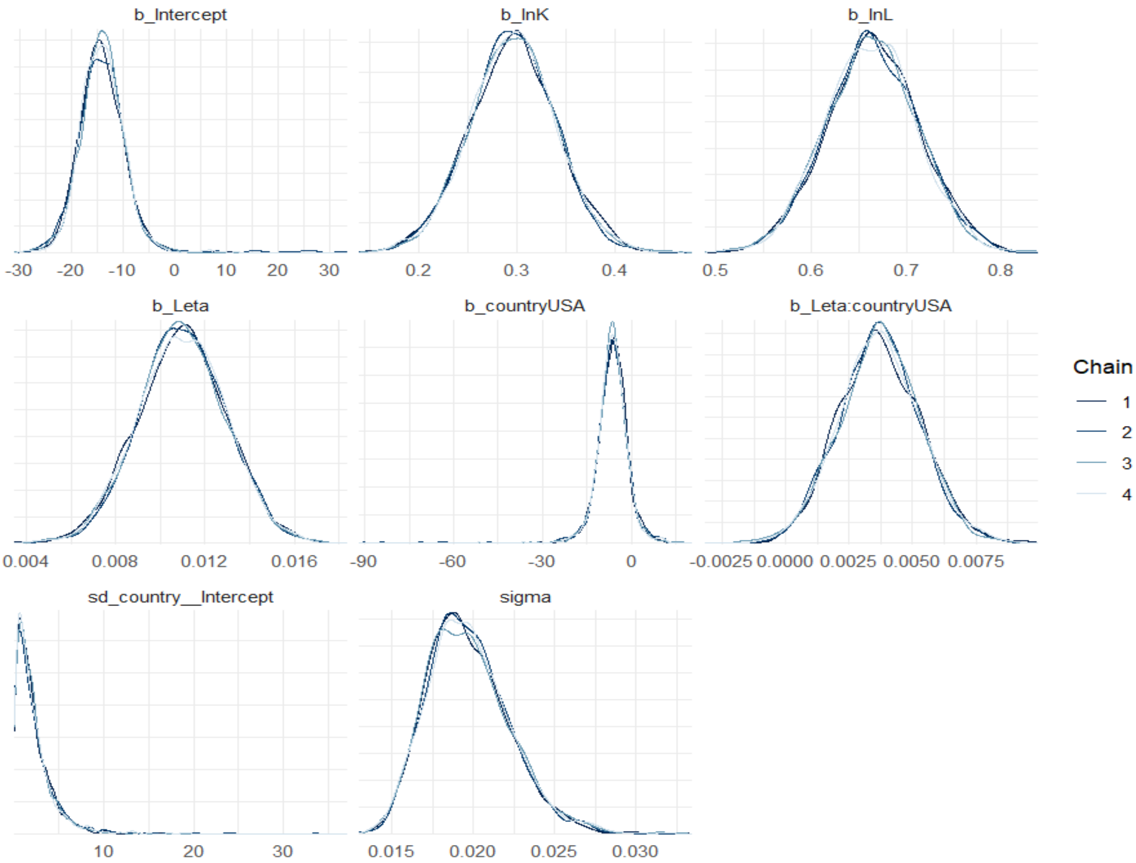

Figure 3: Posterior densities of the parameters of the Bayesian model

These results mean that the model is well identified and stable, with no signs of problematic sampling. This is a necessary condition for interpreting the results as reliable.

The second step of validation concerns the question of whether the model not only explains the existing data well but also adequately predicts their structure. Figure 4 shows the posterior predictive check (PPC), where the actual values (y) are compared with the simulated replications from the posterior distribution (y_rep). The result is an almost perfect match, as the model's simulations closely track the actual observations.

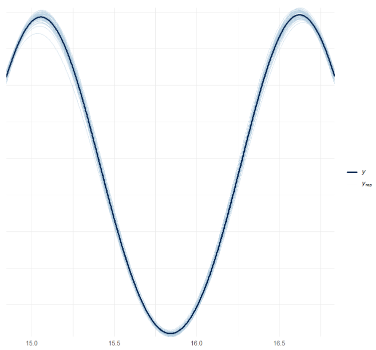

Figure 4: Posterior predictive check – comparison of predictions and actual values

Figures 5 and 6 test whether the model adequately captures the basic moments of the data. Figure 5 shows the distribution of means of the simulated samples, which completely overlap with the actual mean, while Figure 6 shows the same for the standard deviation. In both cases, the actual value lies within the centre of the simulated distribution, meaning that the model exhibits no systematic bias either in central tendency or in data variability.

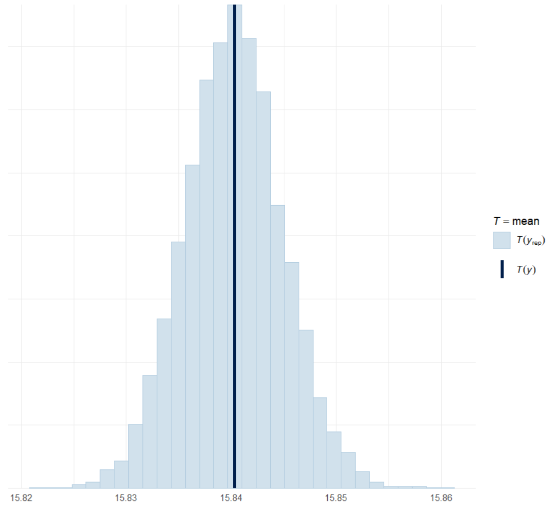

Figure 5: Posterior check of the mean

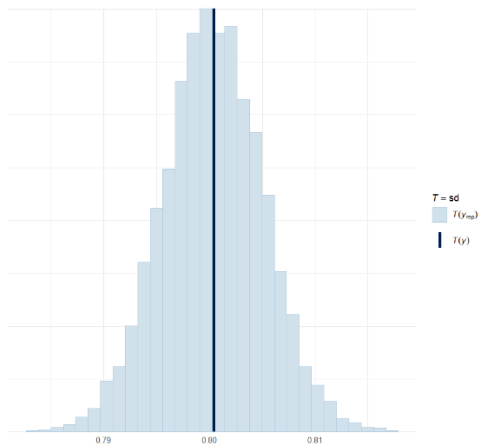

Figure 6: Posterior check of the standard deviation

These results confirm that the Bayesian model not only captures the average effects of capital, labour, and trend but also correctly reproduces the dispersion of the data. This is key evidence of the model's high predictive reliability.

The third set of validation concerns the hierarchical structure of the model and the residual variance. Figure 7 shows the trace plots of the random intercepts for Germany and the U.S. The chains are stable and oscillate around zero, indicating that the initial productivity levels between the two countries are not significantly different.

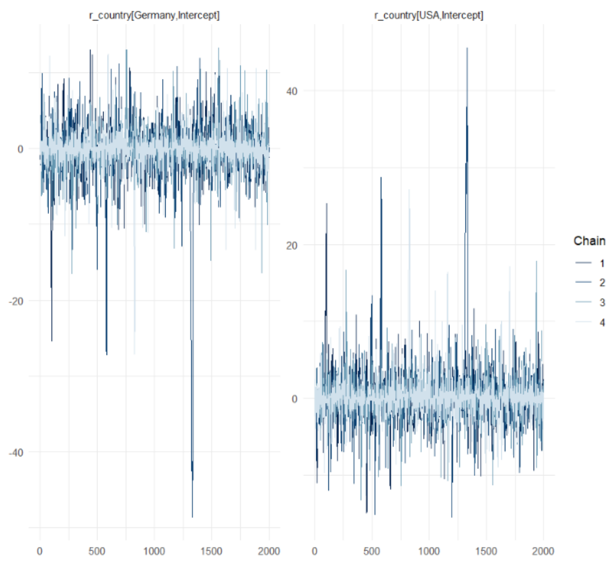

Figure 7: Trace plots of random intercepts for Germany and the U.S.

Figure 8 shows the posterior densities of the intercepts, which are narrow and symmetric around zero. This confirms that the model does not detect systematic differences in the initial level of TFP between the two countries; rather, the differences are expressed primarily in the trend components.

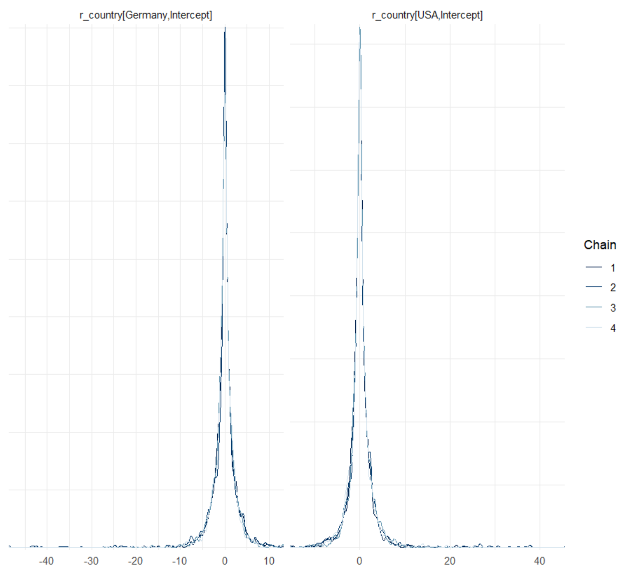

Figure 8: Posterior densities of the random intercepts for Germany and the U.S.

Figure 9 shows the posterior distribution of σ (residual variance). The distribution is unimodal, concentrated around the value 0.02, with a narrow confidence interval. This means that the variance of the residuals is stable and that the model does not underestimate uncertainty.

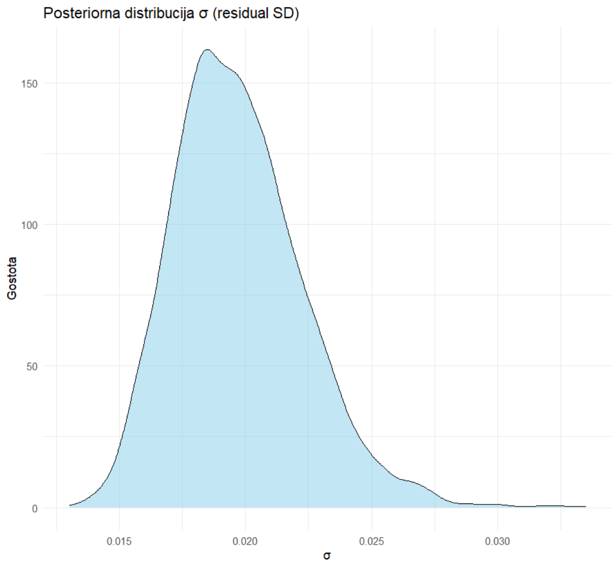

Figure 9: Posterior distribution of the residual variance σ

Taken together, these results show that the key differences between Germany and the U.S. do not stem from initial conditions but from the dynamics of TFP growth. The model thus separates structural initial differences from trend effects, which increases the robustness of the overall analysis.

Robustness Analysis without 2008

To test whether the results are dependent on one-time global shocks, a robustness check was carried out in which the year 2008 was excluded from the sample. The results of the model comparison (with and without the crisis year), summarized in Table 6, show that the main findings remain essentially unchanged.

The comparison of the two models (with and without 2008) indicates that excluding the crisis year does not significantly alter the main results. The difference in the expected log predictive density (elpd_diff) is practically zero (–0.3 with a standard error of 0.4), meaning that the predictive adequacy of the two models does not differ statistically.

Table 6: Comparison of parameter estimates with and without 2008

Parameter	**Estimation (without 2008)**	**95% CI (without 2008)**	**Estimation (full model)**	**95% CI (full model)**
lnK	0.299	0.260 – 0.338	0.295	0.208 – 0.382
lnL	0.652	0.614 – 0.693	0.664	0.569 – 0.759
Years	0.0108	0.0084 – 0.0131	0.0110	0.0071 – 0.0147
countryUSA	–6.464	–16.22 – 3.80	–6.667	–18.09 – 4.19
Years × countryUSA	0.0038	0.0011 – 0.0066	0.0037	0.0006 – 0.0069

The capital and labour coefficients remain very close to the previously estimated values (α ≈ 0.3 for capital and 1–α ≈ 0.7 for labour). The time trend is also stable, with an estimate of approximately 0.011 and a narrow confidence interval. The key difference emerges in the interaction coefficient (Years × countryUSA): without 2008 it increases to 0.0038, with a 95% confidence interval (0.0011 to 0.0066) that is strictly positive. This means that the posterior distribution indicates a statistically significant higher U.S. TFP trend in the period 2000–2017 once the impact of the global shock is excluded.

Interpretatively, this means that the global financial crisis temporarily blurred the difference in productivity trends, as it strongly affected both economies. However, the long-term dynamics confirm a slow but steady strengthening of the U.S. advantage in TFP, consistent with findings in the literature on Germany's lag in technological innovation and the slow convergence of European economies.

4. Conclusion

The analysis of total factor productivity (TFP) for Germany and the U.S. was based on a combination of classical methods (OLS, trend regressions, permutation tests) and modern techniques (Bayesian panel model), with the aim of testing the research hypothesis that TFP trends differ between the two countries. The results of the linear models (OLS) showed that capital and labour take on the expected shares of value added, consistent with the Cobb–Douglas specification ($\alpha \approx 0.3$, $\beta \approx 0.65$). Trend regressions confirmed that TFP increased over time in both countries, while interaction models indicated a statistically significant difference in slopes, thus confirming differences in the dynamics of growth between Germany and the U.S. The results of the permutation tests somewhat moderated this finding. Although the classical interaction tests confirmed a difference between the two countries, the permutation tests produced high p-values (above 0.93), meaning that the observed differences could be the result of random variation. This highlights the limitations of the frequentist approach with a small sample and suggests that differences in TFP dynamics between the countries cannot be confirmed unequivocally.

The Bayesian approach provides a more balanced perspective, as it incorporates both economic-theoretical assumptions and prior empirical evidence into estimation. The posterior distributions confirmed the stability of the estimates for capital and labor and suggested a positive, though moderate, effect of the interaction term (trend × U.S.). This means that TFP growth in the U.S. was on average faster than in Germany, with the posterior distribution of the interaction coefficient mainly lying above zero.

Despite the consistent application of different methods, the analysis has several limitations. First, the observation period is relatively short (2000–2017), which reduces test power and increases estimation uncertainty. Furthermore, the analysis was limited to two countries, so the findings on differences in TFP between Germany and the U.S. cannot be generalised to other advanced economies. The methods used (OLS, permutation tests, Bayesian panel model) provide robust insights, but due to the small sample size and the aggregate nature of the data, they do not capture all the structural factors affecting productivity. These limitations imply that the results should be understood as indicative rather than definitive.

Based on all the analyses conducted, we may conclude that the research hypothesis is partially confirmed. The Bayesian panel model provides the strongest and most reliable evidence, showing that the TFP growth trend was higher in the U.S. than in Germany. Posterior distributions indicate a positive and stable interaction effect, suggesting that the U.S. advantage persisted throughout the observed period. Importantly, the Bayesian framework is particularly well suited for small samples, as it combines information from the data with theoretically grounded priors on factor shares, thereby stabilising parameter estimates and reducing sensitivity to sampling variability.

The findings of this study are broadly consistent with the existing literature, which documents persistently higher TFP growth in the United States compared to European economies, including Germany (van Ark et al., 2003; Inklaar et al., 2008; Calcagnini et al., 2021). While parametric models in this study confirm steeper U.S. productivity trends, the Bayesian approach refines these insights by showing that the difference is positive but moderate once uncertainty and small-sample limitations are accounted for. This resonates with the arguments of Fernald (2014) and Crafts and Mills (2017), who emphasise a general slowdown in productivity growth in advanced economies after 2000, but also supports findings by Comin et al. (2020) and Huo et al. (2023) that methodological refinements are crucial for robust international comparisons. By combining classical and Bayesian methods, this paper demonstrates that the U.S. advantage persisted during 2000–2017, yet without the dramatic widening suggested in some earlier studies. The contribution of this study thus lies in bridging empirical estimates with methodological advances, showing that productivity comparisons remain sensitive to both econometric technique and structural shocks such as the 2008 crisis.

Policy Implications

From a policy perspective, the finding of a slower TFP trend in Germany indicates a structural challenge for sustaining long-term economic growth. While Germany maintains a strong position in manufacturing, the persistent divergence from the United States points to limitations in its capacity to generate productivity gains through innovation and institutional adaptation. This underlines the need for policies that strengthen the determinants of productivity growth. In addition to supporting research and development and accelerating the diffusion of digital technologies, greater attention should be devoted to the structure of financial markets. The German financial system, traditionally oriented toward bank intermediation, provides stability but offers insufficient channels for financing high-risk, innovation-intensive activities. A more developed equity market and a deeper corporate bond market would broaden the range of financing instruments available to firms. Furthermore, a more substantial development of venture capital, combined with an improved framework for attracting foreign venture capital investors, could provide both capital and international expertise necessary for scaling innovative enterprises. By broadening access to diverse sources of long-term finance, Germany would improve resource allocation and strengthen the growth potential of dynamic sectors. Complementary reforms aimed at enhancing labour market mobility would further support the reallocation of labour and capital, thereby mitigating the risk of long-term divergence in productivity trends and reinforcing Germany's competitiveness in the global economy.

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