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Ancient African Number Systems: Exploring Implications for Present-day Equity, Diversity, and Inclusion in Mathematics Classrooms

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Abstract

Inclusive educational systems value the unique contributions students of all backgrounds bring to the mathematics classroom and allow diverse groups to grow side by side, to the benefit of all. Such wide differences are easily catered for when instructional artefacts are culturally curated and utilized for teaching and learning. The differentiation that is present in African Indigenous Knowledge Systems (AIKS), such as ancient numeration systems, has been affirmed to enhance student self-monitoring, and ensures all students are involved, experience success, and yet are still challenged to improve their academic performance. However, many African indigenous numeral systems are already endangered, with some almost extinct, cutting off the present generation of learners from an appreciation of their cultural roots. This reality has necessitated an exploration of ancient African number systems to re-emphasise the need for incorporation into the modern mathematics curriculum for preservation. This study is a conceptual, non-systematic narrative literature review aimed at a broad exploration of the present-day implications of African numeration systems for the mathematics classroom. The study considers the origin of numeration, with particular interest in documented systems across the African continent, including those of Bambara, Bamum, Yoruba, Mende, Oberi Okaime, and Wolof. The implications of these indigenous numeration systems for equity, diversity and inclusion (EDI) in present-day mathematics education were duly considered. It is hoped that the deliberations of this study will spur all concerned stakeholders to undertake a joint venture to revive and preserve our cherished intellectual heritage.

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1. Introduction

Inclusive Education is when a school provides education to children of all abilities and backgrounds. Inclusive schools consist of diverse mathematics learners, including logical learners, visual learners, auditory learners, learners with varied reading or writing preferences, kinaesthetic learners, social and interpersonal learners, and individual and interpersonal learners, to name but a few. Indeed, providing a wide range of learning experiences which promote student participation and engagement is a core ingredient for inclusive education in schools.

The use of the term "inclusion" in education varies across contexts. Göransson and Nilholm (2014) identified four different types of definitions of inclusive education when investigating how it is used in research literature: the placement of students with special educational needs in mainstream classrooms; the social and academic needs of students with special educational needs; the social and academic needs of every student; and creating communities. These varied definitions imply that inclusion is interpreted and used differently, ranging from a strong connection to special education to a community issue, depending on context and cultural diversity (Roos, 2022).

Diversity can be conceptualised in different ways depending on the context in which it is viewed. In the educational context, it encompasses unique differences in race, gender, age, ethnicity, sexual orientation, socio-economic status, ability, religious or political beliefs, or other ideologies of students (Bossu *et al.*, 2019). To cater for present-day diversity, educational providers seek to accommodate these differences in their activities and policies. As regards equity, Bossu et al. (2019) alluded to insight into four basic interpretations of equity. First is the equity of access or equality of opportunity ("Do all individuals or groups of individuals have the same chance of progressing to a particular level in the education system?). Secondly, there is the equity in terms of learning environment or equality of means ("Do all individuals enjoy equivalent learning conditions?"). According to Bossu *et al.* (2019), this question is generally taken to mean: "Do disadvantaged individuals or groups benefit from a learning environment equivalent to advantaged individuals or groups in terms of the level of training of their teachers and other staff, and the quantity and quality of teaching resources and approaches?" In the third interpretation, equity is considered in terms of production or equality of achievement (or results) ("Do students all master, with the same degree of expertise, skills or knowledge designated as goals of the education system?"). Most particularly, Bossu *et al.* (2019) asked whether individuals from different backgrounds achieve equivalent outcomes over the course of education or training. Do all individuals have the same chance of earning the same qualifications when they leave, and can they do so, independent of their circumstances of origin? This concern about equality in achievement is grounded in an ideal of corrective justice and is inevitably accompanied by a desire to narrow the gap between high and low performers from the start to the end of their educational programme (Bossu *et al.,* 2019). Fourthly, there is equity in the use of educational results. Once they have left the education system, do individuals or groups of individuals have the same chances of using their acquired knowledge and skills in employment and wider community life (Bossu *et al.,* 2019)? These four conceptualizations has been applied differently across diverse contexts within mathematics education research.

Schools, in general, and mathematics classrooms, in particular, reflect societal diversity and inequities, and they also offer opportunities to transform them. Providing access to quality mathematics education for learners from diverse cultural perspectives and knowledge backgrounds, including racial, ethnic, religious, linguistic, gender, socio-economic status, and sexual orientations, is an important endeavour. Within this context, it is not surprising that there has been a growing focus on equity-related research in mathematics education practices, theories, curricula and policies (Vithal *et al.*, 2024). The theoretical work of Gutierrez (2012), for instance, argues that equity in mathematics education needs to be conceptualised in relation to access, achievement, identity, and power, all of which are interrelated. Access refers to the resources available to support mathematics learning; achievement refers to student outcomes, which are crucial in determining students' futures; identity refers to the holistic growth of learners as social and cultural beings that mathematics education can and should promote; and power refers to how social relations between different hierarchies play out in mathematics classrooms (Gutierrez, 2012). Consequently, Roos and Bagger (2021) claim that inclusive and equitable mathematics education is an education that strives for every student's opportunity to participate in learning processes and develop the ability and agency to learn.

Within mathematics education as a field of study, Federico Ardila's famous axioms for "Diversity, Equity, Inclusion in Mathematics" could best be used to underscore the need for considering current practices of the mathematical society and pressing for action for quantifiable equity, diversity, and inclusion in mathematics Classrooms (Ardila, nd; De Loera, nd). Axiom 1 states that "mathematical talent is distributed equally among different groups, irrespective of geographic, demographic, and economic boundaries". Axiom 2 states that "everyone can have joyful, meaningful, and empowering mathematical experiences". Axiom 3 holds that "Mathematics is a powerful, malleable tool that can be shaped and used differently by various communities to serve their needs". Axiom 4 states that "every student deserves to be treated with dignity and respect". These postulates allude to the fairness with which all pedagogical plans of action must be dispensed to achieve desirable results. In this regard, equity encompasses the removal of systemic barriers and biases (historical or otherwise) to enact the practice of fair and equitable treatment so that all individuals have equal access to and can benefit from the teaching and learning of mathematics.

Much of the work on inclusive mathematics education attempts to contextualise the teaching and learning of mathematics within the local contexts in which it occurs, as well as to examine the structures within which this teaching and learning take place (Silva *et al.*, 2019). How the broader contexts and policies at the national, state, and district levels impact the teaching and learning of mathematics at specific local sites is an important issue, as is how issues of culture, race, and power intersect with student achievement and learning in mathematics. This significance drills down to the origin and selection of mathematics instructional content, underscoring the need to investigate why certain forms of content (e.g., number systems) are favoured in the curriculum over others, regardless of their relevance to the individual communities that host the educational system.

Recognising individuality among students and the relationship between students' participation in STEM classroom practices and their communities are necessary for effective capacity building of current STEM teachers (Abah, Iji & Chinaka, 2024). STEM teachers are to be retrained to be open-minded about issues of race/ethnicity, gender, socio-economic status, and cultural diversity (Katwibun, 2013). This understanding will enable current STEM teachers to respect and appreciate cultural differences among their students, and to be consciously aware of students' expectations, attitudes, and beliefs, within the ambit of cultural sustainability in education (Abah *et al.*, 2024). Such a critical approach to sustainability will entail a conscious effort to incorporate African indigenous knowledge systems, including African numeration systems, in the education of the African child.

Critical mathematics education has focused on questions of social justice, culture and environmental sustainability (Abah *et al.*, 2024). It requires a form of normative competence on the part of critical stakeholders to understand and reflect on the norms and values that underlie our actions, and to negotiate sustainability values, principles, objectives, and goals in a context of conflicting interests, trade-offs, uncertain knowledge, and contradictions. Abah *et al.* (2024) emphasise strategic competence to collectively develop and implement innovative actions that promote sustainability at the local level and beyond, as well as critical thinking competencies in questioning norms, practices, and opinions; reflecting on one's own values, perceptions, and actions; and taking a stand in the sustainability discourse. Despite the necessity of critically re-evaluating existing practices and the need for cultural humility in exploring systemic cultural injustices, very little has been done to examine why indigenous counting systems are absent in learners' daily interactions with mathematics at the Basic School level.

The "culture as sustainable development" approach defines culture as the basis or core of sustainability, an approach which generates sustainability (Laine, 2016). Culture is utilised to develop a new understanding of the human place in the world and to highlight one's human role as a potential initiator of change. Within this framework, change towards a culturally sustainable way of living is achieved through familiar educational themes with various titles relating to STEM education (Abah *et al.*, 2024). The "culture for sustainable development" approach sees culture as the "glue" that binds the ecological, social, and economic pillars. The approach views culture as having a distinct, independent role in sustainable development. This approach is appealing from the perspectives of educational sciences and education as practice because it highlights themes such as multiculturality, cultural rights, local culture and cultural identity, among others, which are strongly present in core curricula (Laine, 2016). These approaches emphasise that cultural humility incorporates a lifelong commitment to self-evaluation and critique, to redressing power imbalances in pedagogical dynamics, and to developing mutually beneficial, non-paternalistic partnerships with communities on behalf of individuals and defined populations (Tervalon & Murray-García, 1998). They emphasised that cultural humility was a suitable goal necessary in multicultural education (Foronda *et al.*, 2016).

Effective realisation of sustainability requires the implementation of inclusive practices that embrace diversity across the educational, economic, social, cultural, and psychological dimensions. Commenting on the African perspective, Tchombe (2024) emphasises the need to seek alternative solutions to Africa's development challenges, calling for a critical examination of what sustainability and inclusion mean for Africans and how they can impact their lives in the 21st century. Sustainability and inclusion from African perspectives, therefore, raise concerns about the cultural, socioeconomic, political, health, spiritual, environmental, and socially responsible processes, underpinned by Afrocentric humanistic philosophies of communalism and socialism. Inclusion, for Africa, is a mechanism for harmony in human existence, through which the human mind accesses the reality that underlies its actions, practices, and existence. For inclusion to succeed, we need the valuable philosophical principle of togetherness to guide inclusive practices (Tchombe, 2024).

From an African philosophical perspective, belongingness expresses relational unity, translating unity into strength and ensuring a symbiotic relationship for all. The institution of colonialism and the emergence of nation-states put African institutions, customs, and taboos in crisis (Tchombe, 2024). Traditional systems were replaced by commercial interests, alienating Africans to observe persistent tendencies of general disrespect for life and humanity, human rights, freedom, equity, and justice, thus leading to most Africans being unable to lead decent and healthy lives in safe and sustainable environments, with opportunities for economic, educational, social, and cultural development. While Western pedagogical systems often focus on mechanistic, linear approaches, African perspectives are typically more holistic, communal, and interconnected (Zulu, 2006; Abah, 2020). African approaches are shaped by cultural values, spiritual beliefs, and practical experiences that emphasise relationships, interdependence, and sustainability. African systems thinking tends to be holistic, meaning that everything in the world is seen as interconnected. It is not just individual components that are considered, but also the larger whole in which they interact. The African philosophy of Ubuntu, particularly prevalent in Southern Africa, emphasises humanity, interconnectedness, and shared responsibility. The phrase "I am because we are" reflects the communal way of life and suggests that a person's identity and success are inherently tied to the community's well-being (Age, 2024). This philosophy can deeply inform instructional system design by focusing on collective benefit over individual gain.

Historically, African education focused on self-reliant skills, making learners and persons job creators rather than job seekers. The focus was on education, not schooling, which reproduces inequality in all its dimensions and perpetuates exclusion (Tchombe, 2024). In this context, education is all-encompassing and seeks to achieve more than what is available in the formalised schooling system. Integrating all facets of African intellectual development into education entails training citizens who are well-grounded in African cultural values yet open to constructive input from other cultures. Respect for life and human dignity, equal rights and social justice, cultural and social diversity, and a sense of human solidarity and shared responsibility for our common future are important humanistic values that should be integrated into educational systems. This can begin by affording African numeration the dignity it deserves in present-day academic discourse.

Due to the ongoing evolution of the education Africans receive today, numerals have been subject to change, fatalities, and imperilment (Alphonce, 2023). Many teachers and learners cannot count in their local dialect, and when they try, there are obvious inconsistencies (Botlholo, 2025; Sibanda & Tshehla, 2025; Dihangoane, 2020; Sibanda, 2019). As a result, the act of counting using indigenous numerals in a minority language is now left to the community's elders, while the younger generation often shies away from native counting and tends to prefer, adopt, or express numerals in dominant languages (Alphonce, 2023). For Africa, the multiple predicaments of students' mathematical learning, from an ethnomathematical perspective in which issues of power are connected to school mathematical knowledge and its learning, need to be addressed multidimensionally, particularly at the roots (Valero, 2004). These considerations are the contemplation of this study. The deliberations begin by conceptualising numbers and their origins. The study then discusses Africa in the context of number origins before espousing some documented cases of African numeration systems. The study concludes with implications of African number systems for present-day EDI in mathematics education.

2. Literature Review

Number Bases and Their Origins

In mathematics, a number is an abstract entity used for counting, measuring, and performing calculations. Numbers can be classified into different sets based on their properties. The Natural Numbers (ℕ) are the counting numbers: {1, 2, 3, 4, …} (Some definitions include 0 (whole numbers)). Whole Numbers are the natural numbers, including 0: {0, 1, 2, 3, …}. Integers (ℤ) are whole numbers and their negatives: {…, −3, −2, −1, 0, 1, 2, 3, …}. Rational Numbers (ℚ) are numbers that can be expressed as a fraction $\frac{a}{b}$, where a and b are integers and b ≠ 0: $\{\frac{1}{2}, -\frac{3}{4}, 5, -2, \ldots\}$. Irrational Numbers are numbers that cannot be expressed as a fraction, having non-repeating, non-terminating decimals: {π, $\sqrt{2}, e, \ldots$}. Real Numbers (ℝ) are all rational and irrational numbers, representing points on the number line. Complex Numbers (ℂ) are numbers that include a real and an imaginary component, written as: $\{a + bi\}$where *i* is the imaginary unit ($i^2 = -1$). All these types of numbers are fundamental in mathematics and appear in algebra, geometry, calculus, probability, and many other branches of the subject.

However, the notion of number and numeracy must have evolved to the point we have them now. It is evident that Mathematics originated with the practical problem of counting and keeping records. One of the most relevant reasons for teaching Mathematics is considering it an expression of human development, culture, and thought, highlighting its integral role in humankind's cultural heritage (Abah, 2025). Mathematical thinking has been influenced by the vast diversity of human characteristics, such as languages, religions, morals, and economic, social, and political activities. This assertion is very true in the case of number systems. A number system is a way to represent numbers using symbols (digits) and a set of rules; the decimal (base-10) system is the most common, using ten digits (0-9) and a positional value for each digit.

Over the ages, the evolution of numeration systems has expanded human knowledge and abilities for record-keeping, communication, and computation. It is probably not hard to believe that even the earliest humans had some sense of *more* and *less* (Morales & Lippman, 2022). It is evident that the first representation of numbers by symbols came long after humans had learned to count. As culture and civilisation advanced, more efficient and accurate means of calculation and record-keeping became necessary. In a word, the history of numbers is the story of humanity being led by the very nature of the things it learned to do to conceive of needs that could only be satisfied by "number reckoning" (Ifrah, 2000). And to do that, everything and anything was put in service. The tools were approximate, concrete, and empirical ones before becoming abstract and sophisticated, originally imbued with strange mystical and mythological properties, becoming disembodied and generalisable only in the later stages (Ifrah, 2000). The very oldest counting tools that archaeologists have yet dug up are the numerous animal bones found marked with one or more sets of notches. The second concrete counting tool, the hand, is of course even older. A third system has a far from negligible role in the history of arithmetic - the use of pebbles, which really underlies the beginning of calculation. The pebble method is also the direct ancestor of the abacus, a device still in wide use in China, Japan and Eastern Europe. Ifrah (2000) reports that it is the very word "calculation" that sends us back most firmly to the pebble method, considering that in Latin the word for pebble is "calculus".

All societies learned to count their own bodies and to count on their fingers; and the use of pebbles, shells and sticks is universal. So, the fact that knotted string is used in China, Pacific island communities, West Africa, and Amerindian civilisations does not require us to speculate about migrations or long-distance travellers in prehistory (Ifrah, 2000). The making of notches to represent numbers is just as widespread historically and geographically. Since the marking of bone and wood has the same physical requirements and limitations wherever it is done, it is no surprise that the same kinds of lines, "V"s and "X"s are to be seen on bones and pieces of wood found in places as far apart as Europe, Asia, Affica, Oceania and the Americas (Ifrah, 2000). It is therefore not at all surprising that some numbers have almost always been represented by the same figure: 1, for instance, is represented almost universally by a single vertical line; 5 is also very frequently, though slightly less universally, figured by a kind of V in one orientation or another, and 10 by a kind of X or by a horizontal bar.

Until recently, the Fulanis of West Africa used sticks driven into the ground to mark their herds of cattle. Ale (1989) showcases the numerical symbols of the rural Fulanis as follows: 100 is represented by two short sticks placed on the form V, 50 is represented by two sticks placed in the form X, 3 is represented by three sticks III, 6 by IIIIII, 15 by _IIIII (this implies 10 is _, i.e., a stick lying horizontally), 32 by _ _ _II, 54 by XIIII, and 153 by VXIII. With these mathematical symbols, a Fulani man can show, by earth moulding or by placing sticks in front of his house, how many cows, sheep, or goats he possesses. For example, the following was found in the house of a rich cattle owner: VVVVVVXII, showing that he had 652 cows (Ale, 1989). These are not "written" numerals but sticks and stick-crossings fixed into the ground. So, human beings possess, in all places and at all times, a permanent capacity to repeat an invention or discovery already made elsewhere, provided only that the society or individual involved encounters cultural, social, and psychological conditions similar to those that prevailed when the invention was first made (Ifrah, 2000).

Basically, a number is a quantity, an idea which answers the question "how many?" On the other hand, a numeral is a symbol used to represent a number. The history of numerical thought seems to proceed from the discovery of numbers as discrete quantities, through the invention of physical tokens (pebbles, stones, bones, etc.) to represent numbers, to the eventual use of words and symbols to represent numbers. Thus, a numeration system is an attempt to represent numbers using words and symbols, invented differently across cultures. This points to inherent variability in human numerical cognition and arithmetic abilities.

Numerical cognition has been defined to refer to the property of approximation. Malafouris (2010) defined numerical cognition as the capacity for a basic appreciation of changes in quantity and simple number sense observed in both humans and animals. Although present-day readers may take the notion of human progress through numbers for granted, Malafouris (2010) emphasised that it required a significant mental leap to move from counting specific objects (concrete counting) to the abstract concept of number as a representation of quantity. The discrete ordering of the natural numbers (the positive integers) makes them uniquely suited to represent numerosity, that is, countable quantity (Gallistel *et al.*, 2006). This mental leap is unique to humans and sets us apart as an intelligent species.

A *number system* is a way in which humans represent numbers. A number system (or system of numeration) is a writing system for expressing numbers, that is, a mathematical notation for representing numbers of a given set, using digits or other symbols in a consistent manner (Rawat *et al.*, 2012). Given that many cultures and civilisations throughout the ages have developed different ways of using numbers, there are many number systems worldwide. A number system defines a set of values to represent quantity. A system of numeration consists of a set of numerals and a scheme or rule for combining the numerals to represent numbers. As in many languages, numerals form a lexical-semantic group that is especially prone to alignment by analogy due to their closed structure, in which words are associated in a paradigm (Pozdniakov, 2018).

Weibull (2004) clarifies that, in a much broader sense, a *number system* is a set of the many ways in which we, as humans, reason about numbers. We reason about numbers by *talking* about them, so we need a way to represent numbers in speech. We reason about numbers by *writing* about them, so we need a way to represent numbers in writing (text); this representation is known as a *notation*. Furthermore, when reasoning about numbers, we need some sort of *number base*, or *radix* (or scale), which is the fundamental number in relation to which all other numbers stand. In recent times, we have also given a lot of thought to different sets of numbers we have created; therefore, the term number system has also been used to refer to one of these sets (Weibull, 2004). Each of these number concepts is clarified further to provide insights into the existence of numbers.

Numbers in speech were first evident in the usage of number words. Any language generally has number words for a finite set of numbers, e.g., the numbers smaller than the number base of the number system and perhaps certain multiples of the number base, and rules to form larger ones from these (Weibull, 2004). Similarly, *Number notation* is the way in which we form numbers in written text. The notation is often tightly linked with the language being employed, as there is often a connection between the way words are formed in speech and the way they are formed in writing. *Number base* is a system of counting natural numbers in bundles. Some languages have their own unique way of counting numberswhile others have, while others use the same method. For instance, numbers are counted in bundles of ten, called base-ten or denary. As Weibull (2004) observes, a *number base* is a number in relation to which other numbers stand. Numbers are expressed in a given base, and calculations are performed using the values of a number in that base. Throughout the ages, there have been an exceedingly large number of *radices* – a somewhat more scientific term with the same meaning as number base – by which mathematicians have performed their calculations. The number base is of grave importance to the possibility of further advancement of a number system and the mathematics utilising it, considering that a number base can stagnate a whole civilisation's mathematical progression (Weibull, 2004). Finally, *Number sets* are sets of numbers that we can perform mathematical operations upon, as exemplified in the opening paragraph of this section.

In ancient times, in addition to our usual base-10 number system and to systems with a smaller or similar-sized base, some cultures used number systems with much larger bases (Aguilar *et al.*, 2023):

i. Babylonians used the 60-based system. We still divide an hour into 60 minutes, a minute into 60 seconds – this idea originated with the ancient Babylonians.
ii. Ancient Romans used the base-20 system. This can still be traced to how numbers are named in modern French: for example, 80 is *quatre-vingts*, meaning "four-twenties", and 96 is *quatre-vingt-seize*, meaning "four-twenties-sixteen". A similar 20-based system – with 20 divided into four 5s – was used by the Mayans and by the Aztecs.

iii. An unusual 40-based system was used in medieval Russia. For example, to describe the (large) number of churches in medieval Moscow, the Russian chronicle says that there were 40 of 40s (*sorok sorokov*), i.e., $40 \cdot 40 = 1600$.

Ever since the invention of alphabetic writing by the Phoenicians (or at least, by a northwestern Semitic people) in the second millennium BCE, letters have been used for numbers (Ifrah, 2000). The simplicity and ingenuity of the alphabetic system led to it becoming the most widespread form of writing, and the Phoenician scheme is at the root of nearly every alphabet in the world today, from Hebrew to Arabic, from Berber to Hindu, and of course, Greek, which is the basis of our present (Latin) lettering. Given their alphabets, the Greeks, the Jews, the Arabs and many other indigenous peoples thought of writing numbers by using letters. For instance, in the present era, the system assigns numerical values from 1 to 9, then in tens from 10 to 90, then in hundreds, etc., to the letters in their original Phoenician order (an order that has remained remarkably stable over the millennia). Number expressions constructed in this way worked as simple accumulations of the numerical values of the individual letters. The mathematicians of Ancient Greece rationalised their use of letter-numbers within a decimal system, and, by adding diacritic signs to the base numbers, became able to express numbers to several powers of 10 (Ifrah, 2000).

Once they had grasped abstract numbers and learned the subtle distinction between cardinal and ordinal aspects, our ancestors came to adopt a different attitude towards traditional "numbering tools" such as pebbles, shells, sticks, strings of beads, or body parts. Gradually, these simple mapping devices became genuine numerical symbols, which are much better suited to the tasks of assimilating, remembering, distinguishing and combining numbers. Another great step forward was the creation of names for the numbers. This allowed for much greater precision in speech and paved the way for genuine familiarity with the universe of abstract numbers.

There are essentially two types of numeral systems. One is called the unary system. A unary system is a type of numeral system that uses something like tally marks. Each number is represented by a different number of symbols. Another type of numeral system is called a positional system. A positional system uses place value to show the value of numerals (Penn Museum 2013). The Hindu-Arabic system is the positional decimal system that we use. Within a number system, the cardinal and ordinal aspects of numbers describe different properties of numbers. The cardinality of numbers refers to quantity ("how many"), while ordinality refers to the position (ranking) of numbers.

The Hindu-Arabic number system is the most widely used numerical system in the world today, originating from ancient India and later transmitted to Europe through Arabic scholars. It consists of ten digits (0-9) and operates on a base-10 (decimal) system, allowing for efficient calculations and easy representation of large numbers through place value and positional notation. The introduction of zero as a numeral was a significant advancement, enabling complex arithmetic operations and algebraic developments. This system replaced cumbersome numeral systems like Roman numerals due to its simplicity and adaptability, forming the foundation of modern mathematics, science, and commerce.

The place value number system is a mathematical notation in which the position of a digit determines its value, based on a specific base. In the commonly used base-10 (decimal) system, each digit's value is a power of ten, with positions representing ones, tens, hundreds, and so on. This system allows for efficient arithmetic operations and simplifies the representation of large numbers. The concept of place value is fundamental to modern mathematics, enabling the use of zero and facilitating complex calculations (Lande, 2014). It contrasts with non-positional systems like Roman numerals, which lack a structured place-based hierarchy.

Definition: A positional numeral system has a base $b > 1$ and a set of basic symbols for $0,1,2,3,\ldots,b-1$. The *b* basic symbols of a system are called its digits. From this definition, any nonnegative integer N can be written uniquely in the form:

$$N = a_n b^n + a_{n-1} b^{n-1} + \cdots a_2 b^2 + a_1 b + a_0$$

Where $0 \leq a_i \leq b-1$ for each $i \in \{0,1,2,3,\ldots,n\}$.

We then represent *N* with respect to base *b* as the sequence of basic symbols: $a_n a_{n-1} \ldots a_2 a_1 a_0$. Observe that the position of the basic symbol in this sequence determines the power of the base by which it is multiplied. For example, in our base 10 Hindu-Arabic numeral system, the number 247 stands for $2 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$. In base 2, this number is 11110111 because $247 = 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$. Of course, for unambiguity, a positional numeral system requires a symbol for 0.

The concept of zero has a rich and complex history. Early civilisations, like the Babylonians (circa 300 BCE), used a placeholder symbol in their numeral system, but it was not a true zero. The ancient Mayans independently developed a symbol resembling zero around 400 CE. However, the modern idea of zero as both a number and a mathematical concept emerged in India around the 5th century CE, with Indian mathematician Brahmagupta formalising its rules in the 7th century. From there, it spread to the Islamic world, where scholars like Al-Khwarizmi helped refine and disseminate its use. By the late Middle Ages, it reached Europe through translations of Arabic mathematical texts, eventually becoming fundamental to modern arithmetic, algebra, and computing.

Although there is no documented evidence that any ancient African civilisation utilised the positional place value system, there are glimpses of higher-order number bases across the continent. For instance, Abah (2020) reviewed evidence for the vigesimal (base 20) system of the Yoruba and the duodecimal (base 12) system of the NOK civilisation of West Africa. Base 12 is familiar to us in measuring time and in cooking. We often count items such as fruits and eggs in dozens. A dozen dozen is called a *gross*. And there are 12 months in a year. By comparing our modern base-10 (decimal) system with other bases, we will quickly see that the system we are so used to, when slightly changed, will challenge our notions of numbers and the symbols that represent them. The development of number theory to its present form is of great significance for Africa and the entire world. There is an intellectual thread that runs through all technological advances: measurement and calculation. Geometric calculations led to breakthroughs in painting, astronomy, cartography, surveying, and physics. The introduction of mathematics in human affairs led to advancements in accounting, finance, fiscal affairs, demography, and economics – a kind of social mathematics. All reflect an underlying 'calculating paradigm' – the idea that measurement, calculation, and mathematics can be successfully applied to virtually every domain (Malmberg & Malmberg, 2023). This paradigm spread across the globe through education, as seen by the proliferation of mathematics textbooks and schools. It was this paradigm, more than science itself, that drove progress. It was this mathematical revolution, with numeration at its heart, that created modernity (Malmberg & Malmberg, 2023).

Critical Indigenous Pedagogy

Indigenous education centres on the teaching of Indigenous knowledge, methods, and content within both formal and non-formal educational contexts. The recognition and application of Indigenous educational approaches can serve as a response to the erosion and loss of Indigenous knowledge resulting from colonialism, globalisation, and modernity. Through reclaiming and revaluing their languages and cultural traditions, Indigenous communities can strengthen educational outcomes for Indigenous students and support the ongoing survival and continuity of their cultures (Ogbo & Ndubisi, 2021). One theoretical framework for articulating the inclusion of indigenous artefacts, such as ancient numeration systems, in present-day formal education is Critical Indigenous Pedagogy (CIP).

Critical Indigenous Pedagogy (CIP) is an educational approach grounded in Indigenous ways of knowing, being, and doing, while actively challenging the ongoing impacts of colonialism in education. It centres Indigenous voices, experiences, and knowledge systems that have historically been marginalised or excluded from mainstream curricula. Rather than positioning Indigenous knowledge as supplementary, critical Indigenous pedagogy asserts its validity and necessity, questioning dominant Western epistemologies and the power structures that sustain them. In doing so, it reframes education as a space for truth-telling, resistance, and cultural affirmation. At its core, CIP emphasises the revitalisation of Indigenous Knowledge Systems (IKS) and the importance of place-based learning. This means reclaiming traditional languages, oral histories, and spiritualities as valid intellectual foundations rather than mere cultural artifacts. By integrating the "land as teacher," CIP challenges the Eurocentric view of education as a detached, purely cognitive exercise. It seeks to heal the trauma inflicted by colonial schooling, such as the relegation of African numeracy in modern school discourse, by fostering a learning environment where students can reconnect with their ancestral heritage and see their communities' wisdom as the primary source of authority.

Critical Indigenous Pedagogy (CIP) has been shaped by a diverse collective of scholars who sought to fuse the emancipatory goals of Paulo Freire's critical pedagogy with Indigenous sovereignty. Key figures like Sandy Grande, who authored the seminal work *Red Pedagogy*, pushed the field to move beyond Western Marxism toward a framework that addresses the specific "logic of elimination" inherent in settler colonialism. Other influential voices, such as Graham Hingangaroa Smith and Linda Tuhiwai Smith, emerged from the Māori struggle for educational autonomy in New Zealand, providing global models for decolonising research and schooling. Over the late 20th and early 21st centuries, the field developed by shifting the focus from simply "improving" Indigenous test scores to a radical reclamation of epistemic self-determination. This evolution was supported by scholars like Bryan McKinley Jones Brayboy, whose Tribal Critical Race Theory helped formalise CIP by centring the idea that colonisation is endemic to society, requiring an educational response that prioritises indigenous stories as essential theory.

CIP is a multifaceted approach that fundamentally recognises and embeds Indigenous community participation in the development and teaching of Indigenous standpoints and perspectives. While it is strongly concerned with Indigenous perspectives in education, CIP is not limited to this focus, nor is it a fixed product. Instead, it involves deliberate pedagogical choices designed to challenge the narrow frameworks of Western meaning-making within Indigenous studies (Winslett & Phillips, 2005). As a multidimensional, multidirectional pedagogy, CIP supports both independent and collective learning, with understandings developing in a spiral rather than through linear sequences of skill and knowledge acquisition. By centring on a set of core questions continually revisited, discussed, and explored, CIP disrupts assumptions about contingency learning, prerequisite knowledge, and linear progression (Winslett & Phillips, 2005).

Making explicit the knowledge students already hold about Indigenous peoples establishes a personal entry point that encourages learners to take responsibility for their own learning. This prior knowledge is critically examined rather than treated as fixed or inevitable, particularly when the frameworks that have shaped it remain unchallenged. Through Indigenous standpoints on knowledge, these frameworks are disrupted. Employing Indigenous pedagogical approaches both enables and sustains resistance, while creating the freedom for previously unexplored assumptions about Self and Other to surface. This is exactly what re-examining ancient African number systems seeks to achieve.

A key feature of critical Indigenous pedagogy is its emphasis on relationality, responsibility, and community accountability. Learning is understood as a collective, relational process that extends beyond the classroom and is deeply connected to land, language, culture, and ancestry. This pedagogy encourages learners to critically examine their own positionality, assumptions, and privileges, particularly in relation to Indigenous peoples and histories. By interrogating taken-for-granted narratives and prior knowledge, students are invited to engage in transformative learning that disrupts deficit perspectives and colonial logics. As such, Critical Indigenous Pedagogy is inherently transformative and activist-oriented. It encourages students to critically analyse the power structures that continue to marginalise Indigenous communities today, from environmental exploitation to legal disenfranchisement. The goal is to develop warrior-scholars, individuals who possess both the academic skills to navigate the modern world and the cultural groundedness to advocate for their people. By bridging the gap between the classroom and the community, CIP transforms education into a practice of decolonisation, aiming for a future where Indigenous sovereignty is not just recognised but lived.

Critical Indigenous Pedagogy (CIP) is an educational framework that merges the social critique of traditional critical pedagogy with the unique sovereignty and ontological perspectives of Indigenous peoples. Unlike standard progressive models that focus primarily on class or general social justice, CIP centres on the specific history of settler colonialism and the ongoing struggle for self-determination. It argues that education for Indigenous students must go beyond inclusion into the mainstream; instead, it should serve as a tool for "unthinking" colonial narratives that have historically sought to erase Indigenous identities and land connections (Garcia & Shirley, 2012). Ultimately, critical Indigenous pedagogy aims to support Indigenous self-determination and cultural continuity through education. It seeks not only to improve educational outcomes for Indigenous learners but also to reshape educational institutions and practices to be more just and inclusive. Through the deliberate use of Indigenous pedagogical practices, such as storytelling, yarning, and place-based learning, this approach fosters critical consciousness, empowerment, and resistance, creating spaces where Indigenous identities and knowledges can thrive.

3. Data and Methodology

Overview

This study employs a conceptual, non-systematic narrative literature review to explore the present-day implications of African numeration systems for the mathematics classroom. A conceptual non-systematic narrative literature review is a qualitative research methodology designed to provide a broad, interpretive overview of a specific body of knowledge (Cook, 2019). Unlike systematic reviews, which follow rigid, replicable protocols to answer narrow clinical or empirical questions, a narrative review allows the author to synthesise diverse perspectives and "tell a story" about the current state of a field (Ferrari, 2015). This approach is particularly effective for mapping out complex theoretical landscapes, identifying gaps in existing research, and tracing the historical development of a concept. It prioritises the researcher's expert perspective, selecting and critiquing literature based on its relevance to a specific conceptual argument rather than on strict inclusion/exclusion criteria.

The development of this methodology relies on a flexible, iterative process of searching and synthesis. Instead of using a predefined search string across a set number of databases, the researcher often employs "snowballing" techniques, following citations from one seminal work to another, to capture the nuance of a topic. This allows for the inclusion of "grey literature," theoretical essays, and diverse methodological studies that might be filtered out in a more rigid systematic process. The goal is not to provide an exhaustive tally of every study ever conducted, but to distil the most significant contributions into a coherent narrative that clarifies the "what" and "why" of a subject area.

Critically, the value of a non-systematic review lies in its ability to generate new conceptual frameworks and provoke future inquiry. By critiquing the strengths and weaknesses of existing theories, the researcher can highlight contradictions in the literature that a purely data-driven meta-analysis might miss. This makes it an essential tool in the social sciences and humanities, where the objective is often to build a theoretical bridge between disparate ideas. While it is sometimes criticised for potential "selection bias," its strength is its reflexivity; it acknowledges that the researcher's synthesis is an active intellectual contribution that shapes how a field understands itself.

Using a conceptual non-systematic narrative literature review to study ancient African number systems allows one to weave together seemingly disparate fields such as archaeology, ethnomathematics, and modern educational policy into a single, cohesive argument. Because the Ancient African Number Systems archive is often fragmented or oral, a rigid systematic review might fail to capture the necessary depth. This methodology allows the curation of evidence from the Ishango Bone or the Yoruba vigesimal system and directly links it to contemporary theories of "mathematical belonging" for marginalised students.

Rationale for the Narrative Approach

The choice of a narrative methodology is driven by the need for epistemic breadth. To study Ancient African Number Systems, one must look beyond modern mathematics journals into the realms of anthropology and history. By adopting a conceptual framework, this review moves beyond a mere chronological list of discoveries and instead argues how the erasure of these systems contributes to the modern "alienation" of African scholarship in STEM. This methodology facilitates a "thematic conversation" between the past and the present, allowing the researcher to construct a coherent argument for curriculum decolonisation.

Research Questions

In the present study, the key questions are: what is the position of established scholarship on African numeration systems?, what are the various ancient African number systems?, and how can a re-emphasis on existing African numeration systems foster equity, diversity, and inclusion in Mathematics Education?

Strategic Implementation

To execute this study, the researcher organises the narrative around three critical thematic pillars in a blended form without specifying the themes as key headings.

i. The Archaeological-Historical Pillar: The study begins by synthesising literature on ancient African mathematical achievements (e.g., fractal geometry in architecture, archaeological finds of mathematical significance or binary logic in divination). This establishes the "concept" that mathematics is not a uniquely Western invention, challenging the Eurocentric "neutrality" of math.
ii. The Pedagogical Pillar: Here, consideration transitions to modern classroom dynamics. The work reviews literature on diverse stereotypes experienced by indigenous African students when their ancestral contributions are omitted from the curriculum.
iii. The EDI Pillar: Finally, the review synthesises these findings to argue for a "decolonised" curriculum. The narrative shows that Equity, Diversity, and Inclusion (EDI) is not just about "being nice," but about epistemic justice and restoring a suppressed intellectual history to empower students.

Broadly, in a conceptual non-systematic narrative review, the goal is to curate a meaningful "intellectual map" rather than an exhaustive data tally. Because the topic spans ancient history, ethnomathematics, and social justice, the inclusion/exclusion criteria are designed to help select high-impact sources that build a persuasive case for curriculum reform.

The identification of literature was guided by an iterative "snowballing" technique rather than a static search string. This involved starting with seminal texts in Ethnomathematics and Indigenous Pedagogy to identify recurring themes of sovereignty and identity. Literature was selected based on its ability to contribute to the three pillars of this study: the historical validity of African systems, the psychological impact of Eurocentric curricula, and the structural requirements for EDI reform. This selection process ensures that the review is not just a summary of what exists, but a strategic synthesis that challenges the status quo in mathematics education.

Search Keywords

To capture the interdisciplinary nature of the study, the online searches on Google, Google Scholar, ResearchGate, and Bing use a "layered" search strategy based on the study's core pillars. Typical search keywords are listed here.

i. Pillar 1: Ancient Systems & Ethnomathematics
    - "Ancient African mathematics," "Ethnomathematics," "Ishango Bone," "Yoruba number system," "Egyptian geometry," "African fractal geometry," "Binary logic in African divination."
ii. Pillar 2: Educational Frameworks
    - "Culturally Responsive Pedagogy (CRP)," "Critical Indigenous Pedagogy," "Decolonising mathematics," "Epistemic justice in STEM," "Africanized math curriculum."
iii. Pillar 3: Equity & Identity
    - "Math identity," "Mathematical belonging," "Stereotype threat in mathematics," "Equity, Diversity and Inclusion (EDI) in STEM," "Social justice mathematics."

Inclusion Criteria

Since this is a narrative review, inclusion is based on the *conceptual weight* of the source rather than its publication date alone.

i. Seminal Works: Foundational texts in ethnomathematics, even if published before 2015.
ii. Interdisciplinary Literature: Peer-reviewed articles from history, archaeology, and sociology that provide evidence of mathematical thinking in pre-colonial Africa.
iii. Pedagogical Frameworks: Studies or theoretical papers that explicitly discuss the psychological impact of cultural representation on student performance and identity.
iv. Grey Literature: Relevant policy documents or reports from educational organisations (e.g., UNESCO) that define modern EDI standards in mathematics.
v. Diversity of Knowledge: Inclusion of oral traditions or indigenous knowledge systems documented by credible ethnographers.

Exclusion Criteria

These criteria prevent the review from becoming too diluted or straying into irrelevant clinical data. The following kinds of works were excluded.

i. Purely Technical/Abstract Math: Articles focusing on advanced mathematical proofs that do not discuss cultural context or pedagogical application.
ii. Generic EDI Literature: Papers on EDI that do not have a specific focus on mathematics or STEM education.
iii. Non-Contextual Quantitative Studies: Large-scale statistical reports on "achievement gaps" that do not address the historical or cultural reasons for those gaps.
iv. Poorly Sourced History: Non-scholarly or "pop-history" websites that lack archaeological or ethnographic citations regarding African systems.
v. Outdated "Deficit Models": Research that frames Indigenous or African students as "lacking" skills rather than being "underserved" by a colonising curriculum.

4. Results

The outcome of the review is presented here.

Africa and the Origin of Numbers

Mathematics has emerged in Africa in many varied forms over the millennia. Different civilisations have employed sophisticated arrangements of weights and measures, elaborate counting systems, and have played mathematical games. Beyond the North African Islamic and ancient Egyptian civilisations, the history of mathematics in Africa has been little studied, and sources are sketchy (Oxford Mathematics, nd). Most histories of mathematics devote only a few pages to ancient Egypt and to northern Africa during the Middle Ages. Generally, they ignore the history of mathematics in sub-Saharan Africa and give the impression either that this history is not knowable/traceable, or even stranger still, that there was no mathematics at all South of the Sahara (Gerdes, 1996). Some early historians even dismissed the extensive West African numeration systems, including the complex Yoruba system, reflecting the European attitude toward Africans as "primitive savages" (Zaslavsky, 1994). This assertion has been the encore in many historical accounts, even though authors and researchers acknowledge that Mathematics exists in many forms, including language. Academics have commonly repeated stories to students and colleagues that are rarely, if ever, checked for accuracy. Often, this allows mistakes and misinterpretations to creep in through embellishment, as Tuominen (2023) affirmed in an examination of one often-repeated claim about number words in African languages. The casual reader of modern history gets the impression that (outside of Egypt) Africans have largely been incapable of understanding numbers.

Until more archaeological findings confirmed the African origin of mathematics, it was always believed and taught in schools that mathematics originated in Europe. We now know that this is no longer the case, as Africans are, in fact, the first mathematicians (Zulu, 2017). In 1950, Dr Jean Heinzelin of the Belgian Royal Museum of Natural Science in Brussels discovered mathematical artifacts (Heinzelin, 1962) in the fishing village of Ishango, in the present-day Democratic Republic of Congo. This mathematical artifact had a variety of number systems, including the prime numbers, making the Africans the first people to invent prime numbers (Zulu, 2017). It is also reported by other scholars, such as Zaslavsky (1973), that the Ishango Bone was used as a lunar calendar for the periodicity of female menstruation, making the African woman the first mathematician (Zulu, 2017). The Ishango bone is a tool made from a baboon fibula, dated to between 20,000 and 18,000 BCE (See Figure 1). Notches carved into the bone have been interpreted as tally marks, evidence of an arithmetical game, a record of lunar cycles, or simply as grooves to aid grip and therefore entirely non-mathematical (Oxford Mathematics, nd). The latter interpretation has persisted, suggesting that the Ishango bone is controversial evidence of early arithmetic in Africa. Again, the seemingly dismissive attitude toward possible Mathematical prowess in prehistoric sub-Saharan Africa is, at best, a manifestation of neo-colonialism.

Recent research has adequately provided criteria for distinguishing notations from decorated artefacts and suggests several principles for interpreting notations. The ethnographic data triangulated with notch marks indicate that material notations above the range of 10 – 30 marks are hallmarks of trans-egalitarian societies, usually with relatively complex numerical systems that extend into hundreds or thousands (Hayden, 2021). For the Ishango bone, Morales and Lippman (2022) show that the markings on some rows add up to 60, while others contain prime numbers between 10 and 20. As shown in Figure 1, some of the rows appear to illustrate the Egyptian method of doubling and multiplication, and may also represent a lunar phase counter (Morales & Lippman, 2022).

It is easy to believe that there is a general lack of books and research material on the historical foundations of African Mathematics. Most of the "valuable" information on pre-historic Mathematics available concentrates on the eastern hemisphere, with Europe as the central focus. According to Morales and Lippman (2022), the reasons for this may be twofold: first, a lack of specialised mathematics in the unresearched regions; and second, the fact that many of the secrets of ancient Mathematics have been closely guarded. For instance, Chrisomalis (2003) acknowledges that some of the original mathematical scripts from sub-Saharan Africa were developed by religious cults and propagated among closely guarded sects. This reasoning aligns with the concerns about the unique nature of African Indigenous Knowledge Systems (AIKS), as adequately covered in the Advocacy, Human Agency & Collaboration in Decolonisation Working Group (2024) communique.

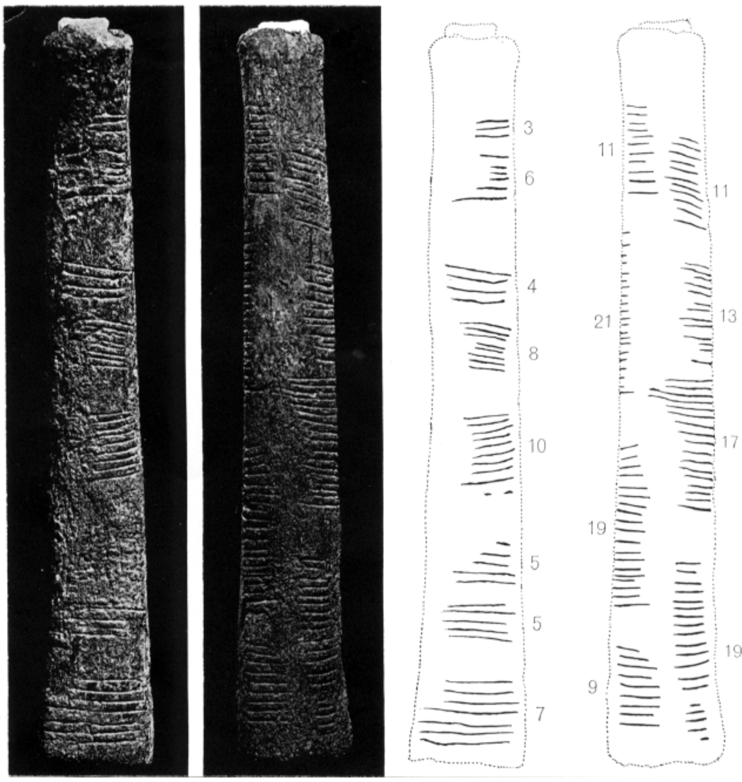

Figure 1: Heinzelin's detailed drawing of the Ishango bone (Source: Heinzelin, 1962)

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Similarly, there is a prevalent claim in the literature examining the history of numbers and the development of number words that some African group ("Bushmen" or "Pygmies") counts in a particular way, where their numerals are of the form 1, 2, 3, 2+2, 2+2+1, etc. (Tuominen, 2023). Numerous forms of this claim are traced to their original sources through Tuominen's (2023) extensive search of the available literature, who affirms that the different forms can be traced to two early sources that have been misquoted and bastardised along the way. Thus, African Mathematical legacy, as it stands in academic literature, is a complicated history of changes and distortions in the sorts of numerals developed and used extensively by our ancestors. Historians have assumed that the complexity of a number system could serve as a proxy for intelligence and thus for the evolutionary development of different peoples, even when their early approaches to indigenous peoples were questionable. The brutal encroachment of pre-existing cultures by colonialism and modernisation eradicated many traces of ancient African numeration, and the current literature in the history of mathematics may draw on problematic older sources (Tuominen, 2023). The rich work of Tuominen (2023) has also shown that the erroneous claims continue to evolve in the literature in various ways. This can take the form of changing the numerals themselves or even wrongly attributing other groups' number symbols, with the error spreading from the academic literature to more popular media like romance novels, movies and Wikipedia (Tuominen, 2023).

The argument around the early African counting systems is often clouded in the treatment of Africa in established historical accounts. Zaslavsky (1994: p.4) recounts the following about her pioneering research for her phenomenal work *Africa Counts*:

> "Books and articles on the history of mathematics were of little value in my research. If they mentioned Africa at all, it was generally to recount the quaint practices of the people who were least advanced mathematically. I must mention two exceptions among pre-World War II publications. A footnote in D E Smith's *History of mathematics* [1923, p. 14] mentioned the work of the Austrian anthropologist Marianne Schmidl (misspelt "Schmidt"), "the standard authority on the number systems in Africa." In her 1915 articles, she warned: "One must be extremely cautious about accepting the accounts of the inability of 'primitive' people to count in higher denominations ('Primitive' must be taken here, as elsewhere, with a grain of salt!)" This was the only instance I found in works from that period where the author placed quotation marks around the word "primitive."
>
> "The other exception is an obscure book by O. F. Raum, *Arithmetic in Africa* [1938]. Dr Raum taught in Tanganyika (now Tanzania) and at Fort Hare University, the only institution of higher learning open to Black South Africans at that time. He described the mathematical practices and games of various African peoples and expressed a strong belief in their mathematical abilities. He stated that good teaching "lays down the importance of understanding the cultural background of the pupil and relating the teaching in school to it" [Raum, 1938, p. 5]."

Thus, Africa is often discredited for its contributions to prehistoric number systems due to a long history of Eurocentric bias in academic and historical narratives. Colonialism and its legacy played a significant role in shaping global perceptions of African civilisations, frequently portraying them as primitive or lacking intellectual sophistication (MacEachern, 2006). As a result, many African innovations, including early mathematical developments, were either ignored or attributed to other cultures. This systematic devaluation of African contributions helped sustain a narrative that positioned Europe and parts of Asia as the sole cradles of advanced knowledge systems. In this regard, the views of Joseph (2011) in his phenomenal book, *The crest of the peacock: Non-European roots of mathematics,* clarifies that a few "mainstream" historians of mathematics have in recent years taken up the task of casting a wider net in writing history and considering seriously the contributions of not only the ancient Egyptian and Mesopotamian civilizations but also the Chinese, Indian, and Islamic civilizations. There are substantial and growing communities of historians of mathematics for all these civilisations, several of whom are working to make new evidence accessible to everyone. Nevertheless, it is argued that change in historical perceptions is slow, and that a significant part of the new studies in the history of mathematics has failed to reach the broader community (Joseph, 2011).

Another reason Africa is discredited is the limited availability of written records from ancient African societies compared to other civilisations. Much of Africa's prehistoric knowledge was preserved through oral traditions rather than written documents, making it more difficult for historians to trace and verify contributions like numerical systems. Additionally, artifacts that do exist – such as the Ishango Bone from Congo, which is believed to represent a form of tallying or proto-mathematics – are often under-analysed or dismissed by mainstream scholarship due to the lack of broader context or written explanation (McIntosh & McIntosh, 1983). Archaeological biases also contribute to the neglect of African prehistoric number systems (Overmann, 2024). Many early archaeological missions in Africa were led by foreign researchers who either overlooked or misunderstood African artifacts due to preconceived notions. Their interpretations often downplayed the sophistication of indigenous technologies and knowledge systems. As a result, discoveries that could point to advanced numerical reasoning were not given the same attention or legitimacy as similar findings in Mesopotamia or Egypt – regions more traditionally associated with the origins of mathematics.

Educational curricula and popular media have historically excluded African achievements in mathematics and science. This exclusion perpetuates the misconception that Africa lacked early intellectual and scientific advancements. The dominance of a Western conception of mathematics has largely been accepted because of the longstanding belief that mathematics originated primarily in Greek and European traditions. This narrative has historically been maintained through tactics such as the omission or appropriation of non-European contributions and by defining mathematics in ways that inherently excluded other traditions. Such a Eurocentric framing has shaped not only the historical account of mathematics but also the broader understanding of it as both a cultural practice and an intellectual pursuit (Joseph, 2011).

Central to this dominant perspective is the portrayal of mathematics as a purely deductive system, ideally founded on axioms and logically unfolding universal truths. This view, though contradicted by ample evidence, has persisted in many earlier histories and is linked to philosophical traditions stemming from figures like Descartes. It fosters an image of mathematics as abstract, timeless, and untouched by practical needs or socio-political influences. In this framework, mathematics is elevated to a realm of pure reasoning, disconnected from material realities and from the diverse ways in which other cultures engaged with mathematical thought (Joseph, 2011).

Contrary to this Western ideal, mathematical traditions from regions such as Africa, India and China often prioritised practical problem-solving and employed a variety of methods to validate results, without relying on formal axiomatic systems. These traditions yielded remarkable insights, such as series summation, the use of triangular number arrays, and the derivation of infinite series, through intuitive, computational, or visual means. However, because these practices did not align with the Eurocentric values of abstraction and deductive rigor, they were frequently dismissed as lacking in sophistication. Reclaiming and valuing non-European mathematics requires a deep re-examination of historical biases, an openness to diverse epistemologies, and a willingness to broaden the definitions of what counts as mathematical knowledge (Joseph, 2011). Efforts to decolonise education and research are beginning to challenge this narrative by highlighting overlooked African contributions, including prehistoric number systems (Ajani & Gamede, 2021). However, until these contributions are fully recognised and integrated into mainstream history, Africa's role in the development of early mathematics will continue to be unfairly discredited.

Documented African Number Systems

Mathematics has *always* existed and has simply been waiting in the wings for humans to discover (Morales & Lippman, 2022). As argued so far, many histories of mathematics adopt quite simple models of the emergence and development of numbers. For example, Seidenberg's diffusion model has been criticised, but some versions of it appear to serve as a basis for many histories (Tuominen, 2023). Many histories can also give the impression that the decimal system is the final form or a linear process of improvement that all number systems are developing towards. Research should be done to see which problematic theories are propagated in these works.

A critical examination of the available literature indicates that Africans have long possessed the faculty for complex numeration systems and have left evidence in a few documented cases. Some of these works are highlighted here.

Mande Numeration System

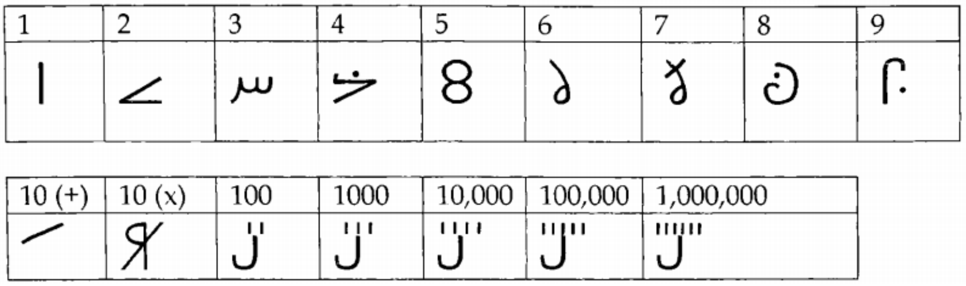
Perekhvalskaya and Vydrin (2019) provided evidence of a sophisticated counting system of the Mande civilisation. The Mande civilisation, a group of ethnolinguistic peoples from West Africa, played a significant role in early West African history. They are credited with the independent development of agriculture and with establishing important empires such as Ghana and Mali. Mande culture is characterised by a dual social structure, a professional class of craft specialists, and a strong sense of self-awareness based on oral traditions. The counting systems and numeral systems found in Mande languages are rather heterogeneous, and some of these systems display unique or at least typologically rare features (Perekhvalskaya & Vydrin, 2019).

Perekhvalskaya and Vydrin (2019) investigated eight numeral systems (old Bamana, modern Bamana, Boko, Dzuungoo, Mwan, Dan-Gwɛɛtaa, San-Maka, and Soninke) that represent identifiable types. Proceed from the principle that "the multiplicand of the lower order, i.e., the radix**,** defines the type of the numerical system", Perekhvalskaya and Vydrin (2019) discussed the following numerical systems exist in the Mande family: decimal (Modern Bamana, Mandinka, Kakabe, Susu, Kpelle, Soninke, Beng, Gban, Yaure, Dan-Gwɛɛtaa, Kla-Dan, Mano, Tura, Goo), pure vigesimal (Seenku, Jalkunan, Jowulu, Guro, Boko), mixed decimal and vigesimal (Vai, Jogo, Ligbi, Bozo-Tigemaxo, Bobo, Mwan, Wan), octogesimal (Old Bamana, Dzuungoo), and mixed decimal and octogesimal (San-Maka).

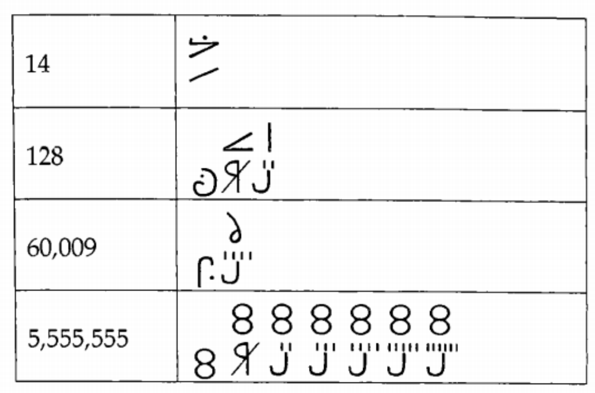
By "mixed decimal and vigesimal", Perekhvalskaya and Vydrin (2019) imply systems where "the numbers up to 99 are expressed vigesimally, but the system then shifts to being decimal for the expression of the hundreds, so that one ends up with expressions of the type "$x100 + y20 + z$". In the mixed decimal and octogesimal system of San-Maka, counting is decimal up to 79 and then switches to octogesimal ($x80 + y10 + z$). Perekhvalskaya and Vydrin (2019) report that the Bozo-Tigemaxo system is peculiar: it is decimal up to 39 and from 100 on, but it is "heteroradical vigesimal" in the intermediate span ($x + 10 + y$, where x is divisible by 20). The same "heteroradical vigesimal" component is also present in both octogesimal systems (Old Bamana, Dzuungoo). Vigesimal models (pure or mixed with decimal ones) do not predominate in Mande, but they are well represented and characteristic of more than a third of the languages (Perekhvalskaya & Vydrin, 2019).

As Perekhvalskaya and Vydrin (2019) observed, the Mande numerals show elements of the quinary model and other models within the first ten, different forms for bases and corresponding numerals, status of numerical bases (higher numerical bases tend to be more similar to nouns, and smaller bases behave more as true numerals), inner syntax of complex numerals (connectors), external morphosyntax of numerals, and some violations of the Greenbergian generalisations concerning subtraction are attested in the Boko system. Greenbergian generalisations refer to linguistic principles or universals proposed by the linguist Joseph Greenberg, particularly in the context of numeral systems. These generalisations, often presented as rules or tendencies, describe recurring patterns observed across various languages and numeral systems, aiming to understand their structure and evolution. The work of Perekhvalskaya and Vydrin (2019) suggests that sporadic non-quinary elements are quite archaic and, as such, constitute real evidence of prehistoric sophistication.

Chrisomalis (2003) discussed a related system of the Mende subculture. The key difference between Mande and Mende culture lies in their geographical distribution, language, and specific cultural practices within the broader West African context. While both belong to the Mande language family, the Mende are a subset of the Mande people, primarily found in Sierra Leone and Liberia, whereas the broader Mande group encompasses numerous ethnic groups across a wider area of West Africa.



i. The Mende Kikakui Numerals



ii. Mende Numeral-Phrases

Figure 2: The Mende Kikakui Numerals (Source: Chrisomalis, 2003)

The *Mende Kikakui Numerals* are credited to the ingenuity of a Sierra Leonean tailor named Kismi Kamara. The system is decimal and multiplicative-additive, with numeral-phrases constructed with the highest exponents on the right. Chrisomalis (2003) noted that because the system is multiplicative-additive, no sign for zero is needed or used. The unit-signs are placed above the corresponding exponent-signs, and so numeral-phrases are read from top to bottom and from right to left. The system is irregular with reference to 10; as such, there are two signs for 10. As seen in Figure 2, the 10(+) form is used additively in combination with units for 1 – 9 to write 11 through 19, while the second 10(x) is the standard multiplicative exponent-sign for 10 used in combination with the unit-signs 2 – 9. This latter form is also used to indicate 10 alone by placing a dot rather than a sign for 1 above it. One unique feature of the *Kikakui* system is the use of vertical strokes to indicate repeated multiplication by 10, where the number of strokes corresponds to the number's power of 10. Chrisomalis (2003) observed that this is quite distinct from the cumulative principle, which always refers to the repeated addition of similar symbols. In principle, Mendes's feat implies that the system could have been extended infinitely without using the positional approach.

The only other interesting multiplicative-additive system in West Africa is the earliest Bamum system from Cameroun (discussed somewhat later). Dismissing any suggestion of acculturation, Chrisomalis (2003) noted that by the time the Mende system was developed in 1921, the Bamum had switched to ciphered-positional numerals. In addition, the use of two different signs for 10 (one additive, one multiplicative) and the use of repeated strokes to indicate successive multiplication by 10 are features that are not attested in other possible ancestral systems. Thus, the structure of the *Kikakui* numeration should be regarded as indigenous (Chrisomalis, 2003).

Kikakui numerals were used for a wide variety of functions and were taught in schools throughout the 1920s and 1930s (Chrisomalis, 2003). Although presently considered extinct, the *Kikakui* system was used for accounting and record-keeping, as well as by craftsmen.

Numeration of the Nok Civilisation

The Nok civilisation was an ancient African culture that flourished in what is now northern Nigeria from roughly 900 BCE to 200 CE. They are known for their advanced terracotta sculptures and mastery of iron smelting, making them among the first societies in Western Africa to transition directly from stone to iron tools. The Nok culture flourished on the Benue Plateau in present-day Nigeria, specifically around the village of Nok. They were a sedentary, mixed-crop farming society that likely engaged in trade. For legacy, the Nok culture is one of the earliest known civilisations in Western Africa and has provided valuable insights into the development of early African societies (Britannica, 2022; National Geographic Society, 2025). One of the earliest sources on the mathematical proficiency of the Nok civilisation is N. W. Thomas' *Duodecimal Base of Numeration.*

Duodecimal Base of Numeration is a treatise published in *MAN*, a monthly record of anthropological science under the direction of the Royal Anthropological Institute of Great Britain and Ireland. In this scholarly work, Thomas (1920) provides a methodical discussion of the facts and principles of the duodecimal (base 12) numeration system of the Nok people in North Central Nigeria. It was a rejoinder to the author's earlier article in the same Journal (1916) and a response to a critique of the earlier article published in the same Journal in 1917. The only point of contention was the critic's argument that *likuru* in Burum numbers stood for ten, and not twelve, as the 1916 article puts it. Responding, Thomas (1920) points "it is possible that both are correct; it happens not infrequently that number words change their meaning when they are passed on from tribe to tribe, and there is no reason why the same thing should not happen within a scattered tribe". Having put the matter to rest, Thomas (1920) launches into an exposition that underscores the importance of documenting the tribes of the North Central region of Nigeria.

The focus of the work is basically on the fact that three languages of the Nok area, *Yasgua*, *Ham*, and *Koro*, are to some extent related, on a base of twelve. Deriving support from a much earlier work by Koelle, Thomas (1920) observes that in the case of Ham, it is expressly stated that they begin again at thirteen, although it is not made clear how the new series is distinguished. Similarly, the numbers in Yasgua up to 24, and other compound numbers, including 20, are based on a base of twelve. The work by Koelle referred to here is the *Polyglotta Africana*, a 1854 study by the German missionary Sigismund Wilhelm Koelle, who based his material on firsthand observations, mostly with freed slaves in Freetown, Sierra Leone.

Thomas (1920) locates the three tribes, Ham, Koro, and Yasgua, as next-door neighbours, with Burum separated from the three by some 120 miles. The number systems of these unique tribes are shown in Table 1.

Table 1: Number System of Yasgua, Koro and Ham (Source: Thomas, 1920)

Number	Yasgua	Koro	Ham
1	*unyi*	*alo*	*jini*
2	*mva*	*abe*	*fali*
3	*ntad*	*adse*	*tat*
4	*nna*	*anar*	*nan*
5	*nto*	*azu*	*to*
6	*ndshi*	*avizi or abirizi*	*toni*
7	*tomva*	*avitar-botar*	*torfo*
8	*tondad*	*anu-oruno*	*nalan*
9	*tola*	*ozakie-othakie*	*mbonkob*
10	*nko(b)*	*ozabe-othabe*	*kowua, kob*
11	*umvi*	*zoelo-uthelo*	*mbonshok*
12	*osog, nsog*	*agowizoe-gowurthue*	*sorho*
13	*nsoi*	*plalo*	*then begin afresh*
14	*nsoava*	*plabe*	
15	*nsoatae*	*pladsie*	

16	*nsoana*	*planar*
17	*nsoata*	*planu*
18	*nsodsi*	*pavizi-eprabithi*
19	*nsotomva*	*plavina-eprovinu*
20	*nsotondad or keuva*	*plarnu-epraruno or zabebe*

From the arrangement of the numerals, Thomas (1920) establishes that Yasgua and Ham form the numbers from five to eight by addition; Yasgua forms nine in the same way, whereas in Ham, nine is clearly subtractive, eight in Ham is additive, and means four (plus) four. However, as Thomas (1920) observes, the situation as regards Koro is not so clear, as six is not necessarily on the base of five, although its outward appearance suggests it. *Abirizi (avizi)* might be explained as *abe-r-adze* (2 x 3). The Koro word for eight, *anu*, may contain the word four *(anar)*, and mean four plus four, but nine, ten, or eleven appear to be subtractive (from twelve), and mean “less three”, “less two”, “less one”, respectively. The form *mbon-shok* (12 - 1) in Ham is clearly parallel to *mbon-kob* (10 - 1), and shows how Ham may have derived this numeral, at least in part, as *shok* is clearly *osog* (12), the form *sorho* is a variant of *sog(h)o*; and twelve appears, therefore, to be the same word in all the language (Thomas, 1920).

What makes Thomas's (1920) work unique is the discovery that no exact parallel to the duodecimal system discussed has been recorded in Africa. In fact, Thomas (1920) asserts that the only other area with a duodecimal base is the Huku-Walengga, North-West of the Lower Semliki (East Africa), and even then, they use this base only for the numbers from twelve to fifteen. It has frequently been assumed that the duodecimal system, found in Europe and crossed with the decimal system, is a product of Babylonia. “But it is clear that, even though Egyptian influence in West Africa may be well established, we can hardly accept such a far-reaching theory as Babylonian influence on numbers below 20, which would surely imply both early and close contact, in the absence of other evidence of Asiatic influence in this area” (Thomas, 1920, p.28). The work *Duodecimal Base of Numeration* added that, evidently, since there is no duodecimal system among any people likely to have been in contact with the Nigerian tribes, it must be admitted that Ham, Koro, and Yasgua (or Yasko) developed the system independently. If it had been transmitted from Babylonia via Egypt, it must surely have left some traces on the road.

Thomas (1920) concluded by presenting a table of the tribes of North Central Nigeria classified according to their numeration system (Table 2). It is important to note that the table reflects the status quo in published resources as of the time the *Duodecimal Base of Numeration* was published.

Table 2: Classification of Tribes based on Numeration System (Source: Thomas, 1920)

Duodecimal	**Uncertain**	**Decimal**
1. Ham	10 . Arago	A. Ankwe
2. Koro	11 . Kagoma	B. Tiwi (Munsi)
3. Yasgua (Yasko)	12 . Agatu	C. Jukun (kororo-fa)
4. Burum	13 . Apu	D. Java (-wa)
5. Ninzam		E. Duggera
6. S. Mada		F. Kagoro
a. N. Mada		
7. Nungu		
8. Mama district		
a. Arum		
b. Barku		
c. Burya		
d. Upye		
9. afu		

An in-depth consideration of Thomas's (1920) results in tonnes of evidence on the geography and knowledge systems of a wide range of tribes in the Middle Belt region of Nigeria. The work also carefully documents the transition of indigenous counting systems from the fingers to cowries, which is probably counted in sixes.

Rwanda-Burundi

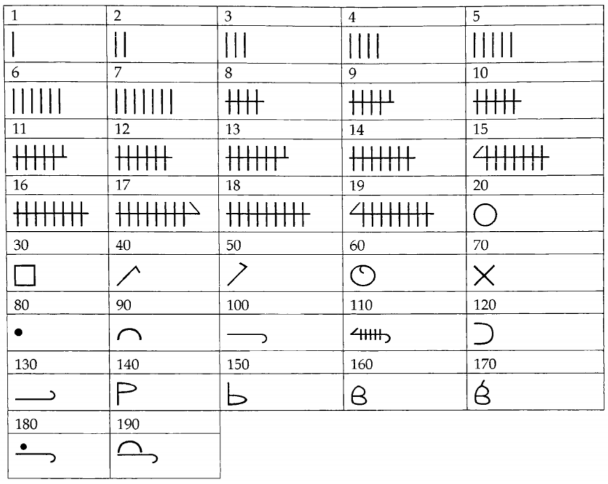
Huylebrouck (1996) reported that counting in the Rwanda-Burundi region was based on the base-10 system, and even for large numbers as large as 1,999,999,999, words existed. As noted earlier, historians do not agree on this point, but mathematicians must admire the feat of inventing words for large numbers. Huylebrouck (1996) observed that there was no written expression, and one can wonder how the slightest arithmetic operations could ever be executed. The *igisoro* board game (a mancala-class board game; see Abah, 2018) was referred to as having been used by the region's civilisation to perform unspoken multiplication, like the convenient Russian peasant method (Huylebrouck, 1996). Evidently, playing with seeds on a piece of wood to solve an arithmetic question may be a diverting and instructive exercise to get an idea of the intellectual effort required to realise a mathematical achievement in a particular cultural environment.

Similarly, original geometric markings on the walls of huts indicate that traditional mathematical concepts were communicated in the Rwanda-Burundi region, not through exchange with other cultures nor as a consequence of the ever-progressing phenomenon of acculturation (Huylebrouck, 1996). The African diversity in number names, gestures and systems (including base 2, probably related to the early Ishango people) shows frequent decompositions of numbers in small groups (like the carvings on the rod), while the existence of words for large numbers illustrates that counting was not merely done for practical reasons (Huylebrouck, 2006).

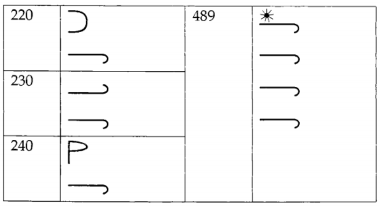
The counting creativity implied that counting in Africa extended beyond purely practical applications, and why some people went to great lengths to count large numbers, tantalising the imagination. According to Huylebrouck (2006), in the language of Rwanda, *10,000* is *inzovu,* or 'an elephant,' and thus *20,000 = inzovu ebyilli*, that is, *two elephants*. Some say the language did not know any larger numerals, but there is evidence that it goes all the way to 100,000, while the Rwandan Abbé Kagame mentions *100,000 = akayovu* or *a small elephant*; *1,000,000 = agahumbi* or *a small thousand*; *10,000,000 = agahumbagiza* or *a small swarming thousand*; *100,000,000 = impyisi* or *a hyena*; and *1,000,000,000 = urukwavu* or *a hare*. The related language of Burundi also goes as far as *100,000 = ibihumbi ijana*, and in the Buganda kingdom, north of Rwanda, greater numerals existed too, such as 10,000,000 (Huylebrouck, 2006).

Bambara

The Bambara of Mali have been attested to have used one of the most peculiar African numerical notations in their religious and divinatory practices (Chrisomalis, 2003). The Bambara system is mostly irregular in structure, and while it is additive, it alternates between cumulative and ciphered notation. Similarly, while it is mainly decimal, it has vigesimal components.



i. Bambara Numeral-Signs



ii. Higher Numeral-Phrases

Figure 3: Bambara Numeration System (Source: Chrisomalis, 2003)

The numbers 1 to 19 are written primarily with vertical cumulative unit-strokes. As can be observed from Figure 3, the value of a set of vertical strokes is doubled if a horizontal line is crossed through it, effectively dividing the number into two registers, one above the line and the other below the line. For odd numbers, an additional half-stroke can be placed at either end of the phrase, sometimes vertically and other times at an angle. Chrisomalis (2003) noted that each of the tens from 20 to 170 has its own sign, which makes the system ciphered at this point. The signs for 180 and 190 are additive combinations of 100 + 80 and 100 + 90, respectively. To add a number of units from 1 to 9 to one of these ciphered signs, an appropriate number of strokes is added to the sign for the multiple of 10, or dots are added when adding units to 60, 160, or 170.

Obviously, the means of representation is decimal if each ten has its own sign to which up to nine unit-signs could be attached. Still, since there are distinct signs for 110, 120 and so on, it is not a ciphered-additive decimal system. Some of the signs for tens are similar enough to those preceding them, e.g., 40 vs 50, 100 vs 110, 140 vs 150, 160 vs 170, suggesting an additional trace of a vigesimal base. For numbers greater than 200, the cumulative principle is again employed by repeating the sign for 100 (another decimal component) as many times as required in a vertical column, with any additional needed signs placed at the top of the column (see Figure 2ii).

Chrisomalis (2003) noted that the Bambara numerical notation system appears to have been used primarily in ritual contexts, especially for divination, although nothing can be said about its origin, period of use, or decline. In terms of ingenuity, the Bambara numerals bear no resemblance to any of the systems that would have been known by the Bambara. As such, it is very likely that the Bambara Numeration System was indigenously invented.

Fula and Wolof

The Fula and Wolof are distinct but interrelated West African ethnic groups, with the Wolof being the dominant ethnic group in Senegal. The Fula are known as pastoralists and are dispersed across many West African countries, while the Wolof are primarily located in Senegal, with a strong presence in urban areas and a history tied to pre-colonial kingdoms. Both groups have a significant cultural influence in the region, and their languages are related. The Fula people speak Fula, also known as Pulaar. The Wolof people are the largest group in Senegal and are primarily located in the western part of the country. Reporting on the numeration system of the Fula and Wolof civilisations, Kosogorova (2023) observed that in Fula, there are different bases for multiplication (multiplicands) and for addition (augends). The Fula numeral system appears, at first sight, to be a three-level system, with three thresholds (lower multiplicands or augends): 5, 10, and 20 (Kosogorova, 2023). That makes the base of the whole numeral system a mixed one – quinary, decimal, and vigesimal.

It is also worth noting that, from a semantic perspective, Pular – like all Fula dialects – employs a *mixed somatic* and *commercial arithmetic base* (Kosogorova, 2023). The somatic base is rooted in natural elements, most commonly involving finger counting, though other body parts or perceptible items may also be used. The commercial base, typically decimal, accommodates larger numbers, making it suitable for trade and economic activities. Additionally, it is significant that the numerical values increase in this system through additive processes. Kosogorova (2023) provided detailed inflectional and derivational possibilities of numerals of Pular Futa-Jallon, Pulaar Futa-Tooro, Fulfulde Maasina, Fulfulde Gombe, Fulfulde Adamawa, and Fulfulde Jamaare, noting original ordinal numerals, fraction numerals, distributive numerals, and human forms of cardinal numerals.

Wolof is another Atlantic language, and, along with Sereer, it is the closest relative to Fula. It is spoken by 10 million people in Senegal and is also the country's lingua franca (the second most widely spoken language after French). There is structural unity in the numeral system across all the dialects, except Lebou, which is well known as a detached dialect (Kosogorova, 2023). The cardinal numerals in Wolof are structured similarly to those of the western dialects of Fula, which are less prone to Arabic influence than the eastern ones and less traditional than the central ones. The first five cardinal numerals constitute a quinary base, which is used to make all the other numerals up to ten. After the first ten, the numeral system of Wolof, just like the one in Fula, acquires traces of a commercial base in addition to the traditional somatic one. Thus, the numeral 'sixteen' is a combination of the lexemes 'ten', 'five' and 'one', and like in Fula 'and'/'with' is placed between tens and units (Kosogorova, 2023). Starting with twenty and up, multiplication is used to express the number of tens, and the units still use the quinary base. Thus, 'twenty-seven' = 2*10 + (5+2).

Kosogorova (2023) reports that ordinal numerals in Wolof have, with one exception, a very uniform internal syntax. An affix *-eel* is added to the last element of a cardinal numeral to turn it into an ordinal one. The only exception from this syntax is, expectedly, the numeral 'one', which has a suppletive ordinal form *(n)jëkk* < *jëkka* 'to be first'. Thus, Kosogorova (2023: p169) concludes that:

> Fula and Wolof are very closely related languages, so originally, they had a similar numeral system. It is a two-stage system with quinary and decimal bases, using multiplication and addition, expressed by the prepositions e for Fula and ak for Wolof (their functions are roughly the same in both languages). The quinary base is presumably of somatic origin; however, this is poorly supported by direct synchronic evidence. The decimal base has a commercial origin.

Although most of the structure of the Fula-Wolof numeral system is unique and original, depicting early African number sense, there is evidence of acculturation. Kosogorova (2023) observed that certain general patterns emerge in the contact phenomena between Fula and Wolof, particularly in their numeral systems. These are summarised in the table below. Wolof is closely related to Pulaar Futa-Tooro due to their shared geographic locations, which explains the comparable patterns of language contact. In the eastern dialects of

Fula, an alternative counting system borrowed from Arabic is used, largely without modification. Central dialects employ distinct systems for counting tens, also derived from loan sources. In Gombe, the system originates from Hausa, while in Maasina, it comes from Bamana - though notably, the Bamana-based system stopped being actively used in the 1960s. Western Fula dialects have long been in contact with Mande languages. This interaction led to the borrowing of a vigesimal base, used specifically for the numeral 'twenty', which was subsequently passed on to eastern dialects. In contrast, Wolof shows relatively limited influence from these contacts, except for borrowed numerals starting from 100 and above. These higher numerals were borrowed from the same languages that influenced the Futa-Tooro dialect of Fula.

Yoruba Numeration System

The Yoruba numeration system is a combination of base-5, base-10 (decimal), and base-20 (vigesimal) systems. It uses distinct words for numbers 1-10, then forms higher numbers through addition, subtraction, and multiples of 20, 100, and 20,000. Despite more recent studies on the uniqueness of the Yoruba numeration system (Ẹkundayọ, 1977; Omachonu, 2012; Babarinde, 2014), evidence from some early scholarly works is presented here to buttress the timelines and originality of the sources.

One of the earliest scholarly works on the indigenous knowledge systems of the Yoruba-speaking people of Nigeria is Adolphus Mann's contribution to the Journal of the Anthropological Institute of Great Britain and Ireland. Surprisingly, *Notes on the Numeral System of the Yoruba Nation* is a linguistic submission that "may interest the student of ethnology and languages and may be of some use in investigating the nature of the mind that can form such an unusual, yet regular structure" (Mann, 1886, p.59). Mann (1886) was drawn to the very different framework of the Yoruba, which has more radical numerals and largely uses subtraction. In the Yoruba numeration system, subtraction is sporadic, used as the third power with addition and multiplication to make up the file of numbers.

Mann (1886) observes that the dynamic numeral system of the Yoruba people is not behind European systems in regularity and symmetry, but surpasses them in the aptitude for interlinking separate numbers. The entire exposition by Mann (1886) buttresses the fact that "for a nation without literature and without a school, knows nothing of abstract numbers" to originate and master such a cumbersome system indicates a high level of mathematical sophistication on the part of the indigenous Yorubas. The work presented superficial knowledge, with a slight attempt at praxis, which sufficed to understand the peculiarities in the arrangement of native numerals, to which analogies in other languages are but rarely found (Mann, 1886).

Another early work, *Yoruba Numerals,* is a classic booklet by Robert G. Armstrong, published by Oxford University Press for the Nigerian Institute of Social and Economic Research in 1962. In the first section (pp. 5-20), Armstrong (1962) analyses the traditional Yoruba numeral system and provides numerous examples. The sequence from one through 130 is given in full; beyond this, several higher numerals – terminating in 1,000,000 – are cited, sufficient to provide a perfectly adequate idea of the nature of the system. Many of the higher numerals are analysed in detail, both in terms of their morphemic components and in terms of the underlying mathematical operations.

The traditional Yoruba numeral system may best be described as "*vigesimal*, with complications" (Wolff, 1963), with the complexity increasing as one proceeds upward beyond 200. The complexity and the resulting unpredictability of the system are the result of several factors (Wolff, 1963):

i. Certain morphemes (e.g., the forms denoting five and 20) have many allomorphs.
ii. Several higher numerals morphemes rather than morpheme sequences.
iii. The patterns of addition and subtraction in the higher numerals differ from those found in the lower reaches of the system, and
iv. The morphonemics of compounding are complex rather than clear-cut.

As Armstrong (1962) points out, the complexity of the system was most probably the result of the expansion of a simpler system for the purpose of accommodating it to the counting of cowries in large-scale trading operations. In this connection, it is worth noting that the modern Yoruba school child, while he receives instruction in Yoruba in the lower grades, rarely learns the system beyond 600. Most educated Yorubas use English numerals in an otherwise Yoruba conversation long before they reach the number 600. The system's complexity is probably a serious obstacle to its use in modern arithmetic operations.

Claudia Zaslavsky is an established authority on the ethnomathematics of Nigerian people and cultures. By 1970, editors of the Two-Year College Mathematics Journal noted that she had been interested in the history of African mathematics for over a decade, describing her as outstanding and perhaps the finest in the field. As noted elsewhere in this deliberation, Zaslavsky's works are revolutionary responses to the suppression of information about the development of indigenous mathematics in Africa in contemporary mathematics textbooks. Zaslavsky's *Mathematics of the Yoruba People and their Neighbours in Southern Nigeria* became one of the earliest voices to proclaim the robustness of African mathematical systems beyond the well-established treatment of ancient Egypt. Zaslavsky (1970) reiterates that, apart from the limited coverage of African mathematics by Western authors, the wide range of superstitions about counting in Africa contributed to the misplacement of ancient mathematical systems on the continent. For instance, there is a widespread fear that counting people and other living creatures will lead to destruction. Therefore, counting is done indirectly by establishing a one-to-one correspondence between the animals and a counting object.

In *Mathematics of the Yoruba People and their Neighbours in Southern Nigeria*, Zaslavsky (1970) refutes earlier Western authors, whose point of view is completely coloured by the prevailing attitude towards Africans as "primitive salvages" deemed hardly human. In cases where these biased authors see the occurrence of numbers up to a million, they dismissed the feat as "remarkable exceptions" to the "law". Such was the extent of the prejudice against dark-skinned peoples that all the principles of logic were turned upside down (Zaslavsky, 1970). There is ample evidence of African's skills in the use of numbers. Part of the culture that each generation transmitted to the succeeding generation was the ability to handle the various kinds of currency then in use. Since a low-valued (cowrie) currency was used as the base, considerable arithmetic skills were necessary to conduct trade outside the confines of the village. Zaslavsky (1970) asserts that, contrary to the generally held European view that Africans could not count beyond ten, the Yoruba could count to a million.

Zaslavsky (1970) confirms that the Yoruba numeration is an example of a vigesimal system, based on twenty, which is widespread in western Africa and elsewhere. The unusual feature of the Yoruba system is that it is subtractive to a very high degree. With numbers that go by tens, five is used as the intermediate figure, five less than the next higher state. In those by 20, ten is used as the intermediate. In those by 200, 100 is used; in those by 2000, 1000 is used. The figure used for calculating indefinite numbers is 20,000 (egbawa), and in money calculation, especially, it is termed *oke kan*, i.e., one bag (of cowries). Large numbers to an indefinite amount are so many "bags" or rather "bags" in so many places. Zaslavsky (1970) provides a table to show words for the first ten numbers in the four principal applications (See Table 3).

Table 3: First Ten Numbers (Source: Zaslavsky, 1970)

Number	Cardinal	Counting	Adjectival	Ordinal
1	*okan*	*ookan-eni*	*kan*	*ekin:ni = ikinini = ako ko*
2	*eji*	*eeji*	*meji*	*ekeji = ikeji*
3	*eta*	*eeta*	*meta*	*eketa = iketa*
4	*erin*	*eerin*	*merin*	*ekerin = ikerin*
5	*arun*	*aarun*	*marun*	*ekarun = ikarun*
6	*efa*	*eefa*	*mefa*	*ekefa = ikefa*
7	*eje*	*eeje*	*meje*	*ekeje = ikeje*
8	*ejo*	*eejo*	*mejo*	*ekejo = ikejo*

9	*esan*	*eesan*	*mesan*	*ekesan = ikesan*
10	*ewa*	*eewa*	*mewaa*	*ekewaa = ikewaa*

The double vowel with which each of the counting numbers begins is a contracted form of the word *owo*, meaning "cowrie" or "money". Zaslavsky (1970) also gives the names of subsequent derivatives as:

11 *ookan laa* (*laa form le ewa* = in addition to ten)
12 *eeji laa*
13 *eeta laa*
14 *eerin laa*
15 *eedogun* (from *arun din ogun* = five reduces twenty)
16 *eerin din logun* (20 - 4)
17 *eeta din logun* (20 - 3)
18 *eeji din logun* (20 - 2)
19 *ookan din logun* (20 - 1)
20 *ogun*
21 *ookan le logun* ("one on twenty" = 20 + 1)
25 *eedoogbon* (30 - 5)
30 *ogbon*
35 *aarun din logoji* (five less than two twenties = (20 x 2) - 5)
40 *ogoji* ("twenty-twos")
50 *aadota* (20 x 3 - 10)
60 *ogota* (3 x 20, *or more* literally "twenty in three ways")
100 *ogorun* = (20 x 5)
105 *aarun din laadofa* (20 x 6 - 10 - 5)
200 *igba*

After 200, the system becomes quite irregular, and the irregularities persist at higher levels. For example:

300 *oodunrun = oodun* [20 x (20 - 5)]
315 *orin din nirinwo odin marun* [400 - (20 x 4) - 5]
400 *irinwo*
2000 *egbewa* (200 x 10)
4000 *egbaaji* (2 x 2000)
20,000 *egbaawaa* (2000 x 10)
40,000 *egbaawaa lonan meji* (ten 2000s in two ways)
1,000,000 *egbeegberun* (idiomatically 1000 x 1000)

Zaslavsky (1970) supports the notion that the pattern in the Yoruba numeration system originated so that one could count the ten fingers of one hand. If the multiples of ten are understood rather than shown by finger gestures, then one finger at a time can be extended to denote 21, 22, 23, and 24, respectively. When the fifth finger is extended, it is subtracted from 30; and when it is retracted, the remaining four fingers are deducted from 30 to give 26, etc.

Zaslavsky (1970) also described an additional set of numerals for counting cowries. This system is based on the cowrie-to-shilling equivalency: 4000 cowries = 1 shilling. The special name for 20,000 cowries is oke kan, meaning 'one bag' or 'five shillings'. Threepence, or *toro*, represents 1,000 cowries, but a penny had a value of only 300 cowries at the time the system was adopted. The subtractive principle, according to Zaslavsky (1970), is used again in the following construction:

Two (shillings) and threepence = *(m)eji le toro*

Two (shillings) and ninepence = *(m)eta din toro* (three less threepence)

Interestingly, the cowrie reckoning is still used today in referring to large numbers. A farmer might say he has "shillings threepence" rather than 5000 yam-heaps. The term "bag" may be used instead of "five shillings".

Zaslavsky (1970) provides an analysis of the relationship of cowrie currency to the numeration systems of the Yoruba. In the northern parts of the region, the cowrie shells were counted out in groups of five, while along the coast, they were pierced, threaded, and generally strung in groups of forty. In areas where cowries were not strung, their use depended upon a rapid method of grouping them in successively higher units. The fact that cowries had to be counted into high denominations by both male and female buyers and sellers certainly should have dispelled the myth that Africans could barely count to ten (Zaslavskey, 1970).

As *Mathematics of the Yoruba People and their Neighbours in Southern Nigeria* relates, the early Yoruba system was based on a combination of vigesimal and decimal counting: 20, 200, 2000 and 20,000 (*oke kan* = one bag). The system at Lagos was later adopted farther north:

40	=	1 string		
2,000	=	1 head	=	50 strings
20,000	=	1 bag	=	10 heads

Zaslavsky (1970) also affirms that in the 1860's the cowrie table and the British equivalents read (with variations depending upon time and place):

40 cowries	=	one string	=	$\frac{3}{4}$to 1 penny
5 strings	=	one bunch	=	3 to 6 pence
10 bunches	=	one hand	=	$1\frac{3}{4}$ to 2 shillings
10 heads	=	one bag	=	14 - 18 shillings

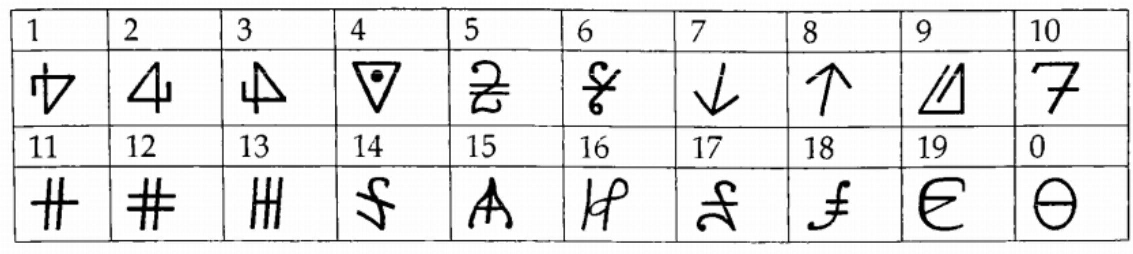
According to Zaslavsky (1970), by the end of the century, one thousand cowries were worth three pence in silver, but a copper penny could be exchanged for only 300 cowries. Additionally:

> During the early nineteenth century, the larger Zanzibar cowries were introduced; European merchants found they could scoop them up by the ton and sell them at a handsome profit. With the added weight of the shells and the depression in value owing to the introduction of European currency, by the end of the century, it was hardly worthwhile to transport them. On the lower Niger River, the cowrie became merely a measure of value – prices were quoted in so many cowrie units. It had become impractical to use them as a medium of exchange except for small purchases in the local markets. The fluctuations in the value of the cowries and the need to convert to British units must have taxed the skills of the shrewdest merchants. As an example, in 1902, the rate was four thousand to a shilling at Ilorin, the northern Yoruba region, while at Sokoto, in northern Nigeria, a shilling would fetch only 1200 shells, since coinage was harder to come by away from the coast. Imports of cowries were banned by proclamation in 1904, and coinage was introduced by the government. However, since the smallest unit was a three-penny piece, cowries continued to be used in local trade until the introduction of the *anini*, worth one-tenth of a penny. (Zaslavsky, 1970. p. 87).

Although the use of cowries as ordinary currency has been discouraged or outlawed, these small shells serve as special-purpose money, as bride price, and for various ceremonial payments. Zaslavsky (1970) attests that most African societies require a material consideration to legitimise and stabilise the marriage, recompense the bride's parents for the loss of their daughter, and guarantee that the husband will fulfil his obligations. Payments of the bride price, which may take several years, allow the man to claim his children; otherwise, they would be retained by the mother's family. If the wife dies without children, or if the couple separates, the bride price must be returned. Europeans might consider it a dowry in reverse (Zaslavsky, 1970). Also, among the Yoruba, ceremonial occasions which require cowrie payments are funerals, initiation into secret societies, and certain fines. As decorations, cowries are seen everywhere - on clothing, drums, divining cowries, head dresses, ritual masks and furniture.

Oberi Okaime

Chrisomalis (2003) explored several alternative numerical notations across different prehistoric and modern civilisations. One of the remarkable examples of scripted mathematical notation from Africa comes from the Oberi Okaime religious sect of the Ibibio-Efik ethnic group in southeastern Nigeria. The divinely inspired leaders of the sect developed an alphabet system written from left to right and a set of numerals. The system is ciphered-positional and vigesimal.



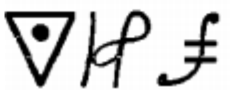
1938 = 

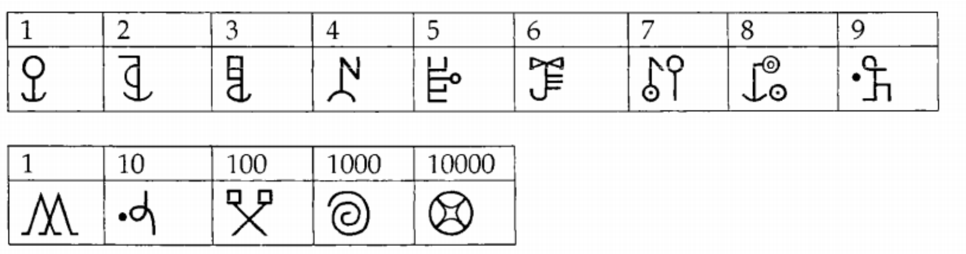
Figure 4: The Oberi Okaime Numerals (Source: Chrisomalis, 2003)

Chrisomalis (2003) affirmed that the Oberi Okaime numeration system was the only known ciphered-positional, base-20 system with no sub-base (with the possible partial exception of the Maya head-glyphed numerals). The Oberi Okaime numeral-phrases are written from left to right with the highest exponents on the left. For instance, 1938 would be formed as $4 \times 400 + 16 \times 20 + 18$ (See Figure 4). The inventors of the Oberi Okaime numerals were educated in Christian missionary schools in the early 20th century, but none of the numeral signs has any graphic resemblance with the corresponding Western numerals except for zero (Chrisomalis, 2003). The numerals were used in relatively few liturgical texts and in confidential correspondence among members of the Oberi Okaime religious sect, a factor that may explain their near extinction in present-day Nigeria.

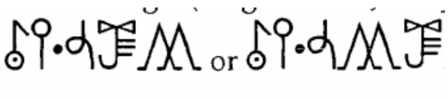
Bamum

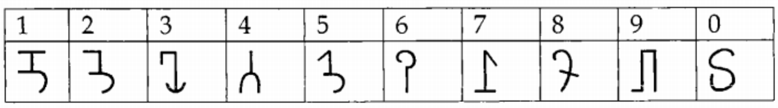
The Bamum of southwestern Cameroun have been credited with creating an extensive numeral system between 1895 and 1903 under the leadership of Njoya, who worked incessantly on the numerals until his death in 1933 (Chrisomalis, 2003). The numbers began with a large logo-syllabary and gradually reduced to number signs, accommodating only 80 characters. The Bamum civilisation used these numerals in the first three decades of the twentieth century, and the system was employed on a variety of legal documents, census records, histories, and personal letters, both printed and handwritten.

The Bamum system is basically decimal and multiplicative-additive, with numeral phrases written from left to right. Surprisingly, the exponent sign for the units could either precede or follow the unit sign (see Figure 5 for an example of 76). The earlier developmental stage of the Bamum numeration system indicated that the system is both a numerical notation system and a set of lexical numerals. Chrisomalis (2003) observed that similar complications arose in the development of the Shang/Zhou and the Chinese classical systems, which are associated with logosyllabic scripts in which some characters (including numeral signs) are ideograms.



i. Original Bamum Numerals

ii. 76 = 



iii. The *mfemfe* Form of the Bamum Numerals

Figure 5: The Bamum Numeration System (Source: Chrisomalis, 2003)

Later refinement of the system reveals that the Bamum scripts have undergone several reductions and simplifications, even though they continue to remain multiplicative-additive. Subsequently, as shown in Figure 5iii, the *mfemfe* form indicated a transformation from multiplicative-additive to ciphered-positional by removing the exponent signs. The old sign for 10 took over the role of 0, and numeral phrases were written from left to right with digits for 0–9, just as in the Western and Arabic positional numerals, which were the Bamum system's main competitors (Chrisomalis, 2003). The system, however, fell into disuse after Njoya was deposed in 1931. Nevertheless, Chrisomalis (2003) noted that the Bamum numerical notation systems, like the scripts, are more than just a historical curiosity, considering their rapid transformation from an additive to a positional structure by the simple step of removing the exponent signs from numeral phrases.

Others

The list of indigenous numeration systems across Africa is inexhaustible, given the emerging scholarship on AIKS by researchers within and outside the continent. Alphonce (2023) discussed the numeral systems of the West Rift Southern Cushitic languages, emphasising that in counting from one to a thousand in Iraqw, Gorwaa, Alagwa, and Burunge, basic numerals are underived, monomorphic, and are used for deriving the secondary numerals through some grammatical and mathematical (addition, multiplication, and combination of) processes. For the indigenous peoples and cultures of Lesotho, Zulu (2017) noted the Basotho origin of mathematics in their use of lunar calendars, their cosmic, empirical, and theoretical knowledge (Nahanotsebo), and their in-depth grasp of mathematical concepts. Similarly, Obikudo (2013) provided details of the numeral system of Nkoroo, an ethnic group in the Niger Delta, whose morphological methods include compounding (for cardinal numerals) and reduplication (for distributive numerals), while the arithmetic methods include multiplication, subtraction and addition. Other related works include Obadan's (2019) consideration of the numeral systems of Ukwunzu and Longuda (Vigeland, 2019), and Ilukena et al.'s (2018) discussion of the historical development of number systems in Namibia.

5. Implications for Equity, Diversity, and Inclusion in Mathematics Classrooms

When scholars write about the history of post-colonial scripts and numerals in Africa, they portray them as derivatives, thereby denigrating Africans' inventiveness. This is an inherent display of inequality and a salient manifestation of coloniality. In a way, those who possess temporal means of power and knowledge consider those who have less as “others”, ripping off the very essence of existence. Researchers of decolonization has referred to this colonial mentality of inequality as “otherness” (Mabvira, 2024; Siahainenia, 2025). While some of the systems already discussed in this exposition (e.g., Mende, Bamum, and Oberi Okaime) are structurally distinct from the Western and Arabic numerals, it is obvious that these systems would not have developed without colonialism and contact with the West (Chrisomalis, 2003). At the same time, however, there is suggestive evidence that pre-colonial Africans (south of the Sahara) also used numerical notations. As granted elsewhere in this discussion, it is not empirically possible to have a full understanding of the numerical notation systems used in sub-Saharan Africa prior to the colonial period.

Instead, researchers have consistently pointed to a handful of ethnographic details about the peoples and cultures of sub-Saharan Africa in the twentieth century, concerning systems that may be considerably older. Since these systems are not directly attached to phonetic scripts, it is difficult to compare them to other numerical notation systems. For ethnographic evidence, Chrisomalis (2003), citing A. S. Judd’s 1917 work, reported that the Munshi people of northern Nigeria (present-day Tiv of Benue State, Nigeria) employed a numerical notation system. This system, which has “a thin line representing the units, a circle for the tens, and a broad line made by the thumb representing a score”, was apparently used when drawing in sand (Chrisomalis, 2003, p.473). Presuming that Judd’s description is accurate, the most likely possibility is that this system was cumulative-additive with a base 20 and a sub-base of 10 (Chrisomalis, 2003).

Similarly, the Dogon of Mali's tradition of graphic symbolism, as seen in rock paintings and sand drawings, includes numerical signs that can be combined. In some symbols, as Chrisomalis (2003) observed, straight lines represent units and circles represent five; a drawing of a man with four circles (each representing one of the limbs with five digits) joined with a cross carries the numerical significance of 22. Another symbol represents a period of sixty years by three rods of decreasing size, each with the value of 20. It appears that, at the time researchers were present, there may not have been a regular correspondence between numbers and written signs, despite the use of cowries for reckoning and calculations.

In the same vein, Chrisomalis (2003) reports that while most systems of tally-sticks use only a one-to-one correspondence, among the Ganda and Djaga (located around present-day Uganda), tally-sticks are also used in which units are marked by small notches, 10 by a larger notch and 100 by an even larger notch. However, it is not clear whether this pre-colonial system is used for recording cardinal numbers, or whether it is simply a series of marks equal to the number being counted, of which the tenth is large and the hundredth larger still.

Surprisingly, most mentions of African indigenous numeration systems are limited to scholarly discussions, with very little knowledge of historic numerical inventions from Africa trickling down to the mathematics classroom where the mathematicians of the future are presently being trained. Even at the higher education level across Africa, treatment of historical numeration systems is limited to ancient Peruvian Quipus, Hieroglyphic representation of numbers, Egyptian Hieratic numeration, the Greek alphabetic numeral system, and Babylonian Cuneiform scripts, without even a footnote on some of the ingenious African scripts discussed so far in the present work. This sounds like a clarion call for efforts geared towards a more inclusive approach to incorporating African cultural heritage into the teaching and learning of mathematics. To embrace diversity and inclusion from a disposition of equality, it would be fair to consider the legacy of intergenerational or structural exclusion and discrimination. Embracing equity, diversity, and inclusion (EDI) means intentionally and actively engaging with communities, individuals, and issues that were previously excluded.

African numeration systems, such as those developed by the Yoruba, Bambara, Oberi Okaime, and other ancient African cultures, have significant implications for promoting EDI in mathematics classrooms. These systems, often overlooked in traditional curricula, challenge the dominance of Western mathematical paradigms and offer an opportunity to validate and celebrate non-Western intellectual traditions. By integrating African numeration systems into teaching, educators can affirm the mathematical contributions of African cultures and promote a more inclusive understanding of mathematics as a global human endeavour (Arinze, 2024). Incorporating African numeration systems fosters equity by ensuring that students from African and African diasporic backgrounds see themselves reflected in the curriculum (Gebre *et al.*, 2021). Representation matters deeply in education; when students see that people from their own heritage have made meaningful contributions to a field like mathematics, it can enhance their sense of belonging and intellectual confidence (Chimakonam, 2013). This approach also addresses historical erasure and counters the narrative that mathematics is exclusively a product of European thought, thereby levelling the playing field for students of diverse backgrounds (Iweuno *et al.*, 2024; Mulaudzi, 2024).

From a diversity perspective, African numeration systems highlight the richness and variety of mathematical thinking across cultures. For instance, the Yoruba base-20 system and the Ndebele counting strategies illustrate that there is no single "correct" way to understand or represent numbers. Presenting students with multiple systems not only deepens conceptual understanding but also cultivates respect for different ways of knowing (Fielding & Makar, 2022). This pluralistic approach reinforces the idea that diversity in thought and experience enhances problem-solving and innovation in mathematics (Claxton, 2005).

Inclusion is further supported when students are engaged in learning environments that validate multiple cultural perspectives. By studying African numeration systems, students can participate in discussions that link mathematics to history, language, and culture, making the subject more accessible and relevant to a broader range of learners (Rovenchak, 2012). Culturally responsive teaching practices that integrate these systems can help dismantle barriers that often marginalise students from underrepresented communities in STEM fields. Ultimately, the inclusion of African numeration systems in mathematics education is a step toward decolonising the curriculum and creating a more just and reflective academic environment (Ajani, 2019). It challenges educators to rethink what is taught and why, and to commit to practices that value all students' cultural backgrounds. In doing so, mathematics classrooms can become spaces where equity, diversity, and inclusion are not just ideals, but lived realities that empower every student to succeed.

Research has shown that the mathematics classrooms in which students learn number facts and number sense through engaging activities that focus on mathematical understanding are preferable to those that promote rote memorisation (Boaler, Williams & Confer, 2015) and that successful mathematics learners are those who can flexibly use numbers by decomposing and recomposing numbers. Introducing African historical numbers should be considered as a key notion in students' participation in mathematical instruction to develop *individual intellectual agency*. Such agency is defined as the learner's individual initiative and ownership of ideas to define, redefine, build, take risks and go beyond the specificities of a mathematical problem (Brown, 2020; Boaler & Greeno, 2000). The concept of agency is bound to the particularities of the context defined by the mathematical problems through which research will invite students to display and build their intellectual activity. This notion of agency focuses on the characteristics of those students as learning cognitive subjects engaged in mathematical activity.

Touching on African number systems is beautiful in its simplicity and challenging in its implications. For instance, Armstrong (1962) reports that a Yoruba informant was greatly impressed by expositions of the possible uses of the Yoruba numeral system and readily agreed that, for present-day purposes, it is vastly superior to the traditional system. The challenge presented by this foregoing claim is of special interest to the student of innovation. In this regard two questions seem of special interest: (1) What are the chances of some unearthed African numeral system gaining official acceptance to the point of permitting its introduction on the elementary school level?; (2) Assuming that it will find such official acceptance, what are the chances of its spread throughout society at a time when English is the official language across most of Africa and the Hindu-Arabic numerals are the international standard? These are obvious concerns that warrant further investigation by scholars of AIKS. The goal, obviously, would not be to unseat the existing system, but to at least acknowledge the ingenuity of indigenous systems on the African continent.

Ancient African numeration systems expounded in this study so far offer a profound pedagogical shift by reframing mathematics as a universal human endeavour rather than a strictly Eurocentric invention. Integrating these systems into contemporary classrooms promotes ethnomathematics, which validates a student's cultural identity while dismantling the "math anxiety" often rooted in the belief that mathematics is an abstract, alien construct. By exploring how ancient Africans used doubling and halving for multiplication (the binary precursor to modern computing), educators can demonstrate that mathematical logic is diverse and adaptable, fostering a more inclusive and globalised perspective on the history of science (Zaslavsky, 1994). From a didactic standpoint, these systems provide unique cognitive tools for developing number sense and mental flexibility. Grounding teacher education in readily available cultural artefacts has been canvassed in research from across Africa (Ogunniyi, 2004; Ogegbo & Ramnarain, 2024). For instance, the additive and subtractive nature of the Yoruba system, where a number like 45 might be expressed as "five from ten from sixty", forces students to engage with decomposing numbers and understanding relative values rather than relying on rote memorisation. This encourages a deeper grasp of place value and base structures, challenging students to think fluidly across different numerical frameworks. Such exercises mirror modern algebraic thinking, as they require constant manipulation of terms to reach a final value (Abah, 2017). Simplifying mathematical abstraction via the use of African numeration systems bridges existing gaps in mathematical reasoning, particularly for learners from rural, under-resourced schools (Owuor, 2007; Onyewuchi & Owolabi, 2022).

Evidently, the application of African numeration systems in the classroom bridges the gap between abstract theory and tactile, algorithmic practice. The use of tools like the abacus, as well as the mathematical patterns found in traditional Sona sand drawings and in fractals in village layouts, provides a visual and kinesthetic way to learn geometry and logic (Abah *et al.*, 2024). These methods emphasise that mathematics is a tool for solving practical problems, from trade to architecture, rather than just a series of symbols on a page. By grounding contemporary curriculum in these ancient yet sophisticated logics, educators can cultivate a more robust, multi-dimensional problem-solving toolkit in their students (Sun *et al.*, 2018).

6. Conclusion

Integrating African numeration systems is not just about adding cultural content – it should be a transformative act that challenges the monocultural nature of traditional mathematics education. It opens doors to equity by validating diverse ways of knowing, encourages inclusion by representing all learners, and fosters a deeper and more human-centred understanding of mathematics.

Integrating African numeration systems into mathematics classrooms has profound implications for promoting equity, diversity, and inclusion. These systems offer culturally relevant entry points that validate the lived experiences and identities of African and African-descended students. By showcasing diverse mathematical traditions, educators challenge the dominance of Eurocentric narratives in mathematics, foster global awareness, and broaden students' understanding of numerical concepts. This not only affirms the contributions of African civilisations to mathematical thought but also creates more inclusive and engaging learning environments where all students can see themselves reflected in the curriculum.

Using African numeration systems supports equity by diversifying pedagogical approaches and enabling differentiated learning. For students who may struggle with numerical methods, these alternative systems can serve as bridges to deeper understanding. Culturally responsive teaching that includes African numeration fosters critical thinking and enhances conceptual grasp by encouraging students to compare and analyse different ways of representing numbers. To implement this effectively, educators must receive professional development and curricular resources that equip them to teach with accuracy and respect. Ultimately, integrating African mathematical knowledge contributes to a more just, inclusive, and intellectually rich mathematics education.

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